جامعة بجاية
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UNIVERSITÉ ABDERRAHMANE MIRA-BEJAIA
FACULTÉ DES SCIENCES EXACTES
DÉPARTEMENT DE MATHÉMATIQUES
LABORATOIRE DE MATHÉMATIQUES APPLIQUÉES

Laboratoire de Mathématiques Appliquées
UNIVERSITÉ de BEJAIA

ACDSODE'21

African Conference on Dynamical Systems and Ordinary Differential Equations

March 20-23

Bejaia University Algeria

Plenary Speakers

Pr El Hadi Ait Dads, Cadi Ayyad University, Morroco.
Pr Svetlin Georgiev, Sofia University, Bulgaria.
Pr. Jarkko Kari , Turku University, Finland.
Pr Mohsen Miraoui, IPEI Kairouan. Tunisia.
Pr J. Leonel Rocha Instituto Politécnico de Lisboa. Portugal
Pr Karim Yadi, Tlemcen University, Algeria.
Dr Maryam Hosseini, IMPAM Isphahan, Iran.

Conference Topics

Two sessions are planned
Session 1 : Ordinary differential equations and continuous dynamical systems
Session 2 : Discrete dynamical systems

Contact :
Website : https://asdcod21.sciencesconf.org

African Conference on Dynamical Systems and Ordinary Differential Equations

ACDSODE 21 March 20-23 2021

Laboratory of Applied Mathematics
Bejaia University, Algeria

African Conference on Dynamical Systems and Ordinary Differential Equations
ACDSODE 21 March 20-23 2021
Bejaia University, Algeria

The conference aims to regroup African researchers working in the field of dynamical systems in a broad sense. This includes a wide range of research directions, Stability of ordinary differential, discrete dynamical systems, chaos, bifurcations, topological and symbolic dynamics and their applications that are at least as rich and diverse.

The academic program of the conference will consist of invited talks and paper presentations. Two sessions are planned, the first one for Ordinary Differential Equations and Continuous Dynamical Systems (ODE/CDS) and the second for Discrete Dynamical systems (DDS).

The organizers hope that ACDSODE will become a regular event in the continent and will help to create links between African mathematicians working in this exciting field. Due to the Covid 19 pandemic this edition will be held online.

Plenary Speakers

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Conference Topics

Two sessions are planned:

Session 1 : Ordinary differential equations and continuous dynamical systems.

Session 2 : Discrete dynamical systems.

Scientific committee

Pr. Nourredine Akroune

Pr. Ahmed Berboucha

Pr. Karima Mebarki

Dr. Fatiha Boulahia

Dr. Nadia Mohdeb

Organizing committee

Pr. Ahmed Berboucha

Dr. Leila Baiche

Dr. Mohand Bouraine

Dr. Rezki Chemlal

Dr. Hacene Gharout

African Conference on Dynamical Systems and Ordinary Differential Equations
ACDSODE 21 March 20-23 2021
Bejaia University, Algeria

Program

Session 1 : ODE / Continous dynamical systems

Session 2 : Discrete dynamical systems.

Saturday 20/03/2021

Session ODE/ CDS: Chairman N. Mohdeb

09h00-09h30: Opening

09h30-10h30: Yadi Karim. University of Tlemcen, Algeria.

"Sur les modèles de prédation avec taux de disparition non constant."

10h30-11h00: Tinhinane Meziani. University of Bejaia, Algeria.

" Etude de la dynamique d'un modèle biomathématique régi par un système d'équations différentielles ordinaires. "

11h00-11h30: Houdeifa Melki. University of BadjiMokhtar Annaba, Algeria.

"Limit cycles for a class of generalized kukles differential systems. "

11h30-12h00: Abdessalem Benterki. University of Medea, Algeria.

"Study the seiqrdp model of covid-19 in Algeria. "

Break

Session ODE/ CDS : Chairman K.Mebarki

13h30-14h30: Svetlin Georgiev. Sofia university, Bulgaria.

" Applications of the Fixed point Theory."

14h30-15h00: Zouaoui Bekri University of Oran 1, Algeria.

"Existence of solution for a third-order boundary value problem. "

Break

15h30-16h00: Ghendir Aoun Abdellatif. Faculty of Exact Sciences, Hamma Lakhdar University, Algeria.
"Nonlocal Integro-Differential Boundary Value Problem for Fractional Differential Equation on An Interval Infinite. "
16h00-16h30: Safa Chouaf. University of 20 August 1955 Skikda, Algeria.
"New Results on positive bounded solutions of a second-order iterative functional differential equation. "
16h30-17h00: Rabah Belbaki. ENS Kouba, Algiers, Algeria.
"On the monotone generalized non expansive mapping in Banach spaces. "
17h00-17h30: Ibtissem Merzoug. University Badji Mokhter Annaba, Algeria.
"Existence of solutions for a nonlinear fractional p-Laplacian boundary value problem. "
17h30-18h00: Moussa Haoues. Souk-Ahras University , Algeria.
"Existence and uniqueness of solutions for the nonlinear fractional differential equations with nonlocal conditions. "

Sunday 21/03 /2021
Session DDS : Chairman N.Akroune

09h00-10h00: J. Leonel Rocha. Polytechnic Institute of Lisbon, Portugal.
"Lambert W function in the stability and bifurcation analysis of homographic Ricker maps."
10h00-10h30: Faiza Zaamoune. University Mohamed Khider, Biskra, Algeria.
" Discovering Hidden Bifurcation in Chua system Via Transformation. "
10h30-11h00: Nouressadat Touafek. Mohamed Seddik Ben Yahia University, Jijel, Algeria.
" On the behavior of the solutions of a system of difference equations of second order defined by homogeneous functions. "
11h00-11h30: Hacene Gharout. University of Bejaia, Algeria.
" Evolution of a three-dimensional endomorphism towards hyper chaos "
11h30-12h00: Bououden Rabah. Abdelhafid Boussouf University Center, Mila, Algeria. " Chaos in the Fractional Lozi Map. "
Break

13h30-14h00: Nouar Chorfi. University of Tebessa, Algeria.

"Stability analysis and an optimal control applied to the spread of HIV/AIDS model."

14h00-14h30: Yahiaoui Yaniss. University of Bejaia, Algeria.

" Sur les bifurcations d'un certain système dynamique discret. "

Break

Session ODE/CDS: Chairman A. Berboucha

15h00-15h30: Nasri Akila. University of Bejaia, Algeria.

" La reduction de R : Smith et application".

16h00-16h30: Kamel Ali Khelil. University of 8 May 1945 Guelma, Algeria.

"On the stability of certain nonlinear delay dynamic equations. "

16h30-17h00: Samir Cherief. University Abdelhamid Ibn Badis of Mostaganem, Algeria.

"Growth of solutions of a class of linear differential equations near a singular point. "

17h00-17h30: Bouharket Benaissa. University of Tiaret, Algeria.

"New proof of Hardy dynamic Integral Inequality on Time Scales. "

17h30-18h00: Amira Ayari. University of Badji Mokhtar Annaba, Algeria.

"Sufficient conditions for exponential stability of some nonlinearperturbed system on time scales."

Monday 22/03/2021
Session ODE/CDS : Chairman F. Boulahia

09h00-10h00: El Hadi Ait Dads. University Cadi Ayyad, MOROCCO.

"Discrete Pseudo Almost Periodic Solutions for Some Difference Equations. "

10h00-10h30: Mesbah Chebbab. University of Tizi Ouzou, Algeria.

"Pseudo almost periodic solution for the Nicholson Blowflies model with Stepanov pseudo almost periodic coefficients. "

10h30-11h00: Mohamed Abdelhak Kara. University Abdelhamid Ibn Badis of Mostaganem, Algeria.

"Fast growing and fixed points of solutions of complex linear differential equations. "

11h00-11h30: Fayal Bouchelaghem. University 8 Mai 1945 of Guelma, Algeria.

”Existence of positive solutions for dynamic equations on time scales. ”

11h30-12h00: Benadouane Sabah. University of Bordj Bou Arreridj, Algeria.

”Explicit non-algebraic limit cycle of a family of polynomial differential systems of degree even. ”

Break

13h30-14h30: Mohsen Miraoui. University of Kairouan, Tunisia.

”On the integro-differential equations with reflection. ”

14h30-15h00: Omar Benniche. Djilali Bounaama University, Ain Defla, Algeria.

” Null–controllability for systems governed by fully nonlinear differential equations. ”

Break

15h30-16h00: Rebiha Benterki. Bordj Bou Arréridj University, Algeria.

” Limit cycles of a family of discontinuous piecewise linear differential systems separated by conics. ”

16h00-16h30: Rachid Boukoucha. University of Bejaia, Algeria.

”Algebraic and non-algebraic limit cycles of a family of planar differential systems.”

16h30-17h00: Saad Eddine Hamizi. University of Bejaia, Algeria.

”A class of planar differential systems with explicit expression for two limit cycles.”

17h00-17h30: Mouna Yahiaoui. University of Bejaia, Algeria.

”Invariant algebraic curves and the first integral of a class of Kolmogorov systems.”

17h30-18h00: Ahlam Belfar. Bordj Bou Arréridj University, Algeria.

”Global Phase Portraits of some Quadratic systems having reducible invariant curve.”

Tuesday 23/03/2021

Session DDS : Chairman R.Chemlal

09h00-10h00: Jarkko Kari. Turku University, Finland.

”Decidability in Group Cellular Automata.”

10h00-10h30: Rezki Chemlal. University of Bejaia , Algeria.

”Periodicity and factors of endomorphisms of the shift.”

10h30-11h00: Saliha Djenaoui. University of 8 May 1945, Algeria.
"Overview of the generic limit set."

11h30-12h00: Tarek Sellami. University of Sfax, Tunisia.
"Common dynamics of Rauzy fractals with the same incidence matrix."

Break

13h30-14h30: Maryam Hosseini. Institute for Research in Fundamental Sciences (IPM), Iran.
"Topological rank of cantor factors of cantor minimal systems. "

14h30-15h00: Aymen Hadj Salem. Higher Institute of Management of Gabés, Tunisia.
"On recurrence in dendrite flows."

Break

15h30-16h00: Mohammed Salah Abd Elouahab. Center Abdelhafid Boussouf, Mila, Algeria.
"On some stability conditions for fractional-order dynamical systems of order alpha in $[0, 2)$ and their applications to some population dynamic models."

16h00-16h30: Hamdi Brahim. University of Mostaganem, Algeria.
"Etude d'un problème à conditions aux limites non locales généralisées de type Bitsadze-Samarskii dans les espaces Lp. "

16h30-17h00: Karima Ait-Mahiout. Higher Normal School, Algeria.
"Solutions multiples pour un problème aux limites poste sur la demi-droite réelle par la théorie de Morse. "

17h00-17h30: Smaïl Kaouache. Centre universitaire de Mila, Algeria.
"Chaos and mixed combination synchronization of three identical fractional hyperchaotic systems with different fractional-order. "

17h30 Closing

Conferences

Plenary Speakers :

1. Pr. El Hadi Ait Dads. Cadi Ayyad University, Morocco.
2. Pr. Svetlin Georgiev. Sofia University, Bulgaria.
3. Dr. Maryam Hosseini. IMPAM Isphahan, Iran.
4. Pr. Jarkko Kari. Turku University, Finland.
5. Pr. Mohsen Miraoui. IPEI Kairouan, Tunisia.
6. Pr. J. Leonel Rocha. Instituto Politécnico de Lisboa, Portugal
7. Pr. Karim Yadi. Tlemcen University, Algeria.

Sur les modèles de prédation avec taux de disparition non constant

A. Hammoum, K. Yadi and T. Sari

Nous proposons l'étude d'un modèle proie-prédateur général dans lequel le taux de disparition est une fonction dépendant de la densité des espèces. Il s'agit aussi d'une tude comparative avec les modèles existant dans la littrature en prcisant dans quelle mesure notre modèle les contient et ce qu'il apporte de nouveau. Nous mettrons en vidence la possibilit d'avoir une bifurcation de Hopf et une discussion est mene sur le paradoxe de l'enrichissement du milieu. Nous dirons deux mots sur ce type de modèle lorsque le rendement est considr comme très petit.

Mots clès: Modèle proie-prédateur, Mortalité non constante, Bifurcation de Hopf, Paradoxe de l'enrichissement.

On the integro-differential equations with reflection

Mohsen Miraoui

Maitre de conférences en Mathématiques Universit de Kairouan

Abstract: By developing important properties on the composition of functions with reflection, using some exponential dichotomy properties and an application of the fixed point theorem, several new sufficient conditions for the existence and the uniqueness of an pseudo almost automorphic solutions with measure for some general type reflection integro differential equations. We suppose that the nonlinear part is measure pseudo almost automorphic and in which we distinguish the two constant and variable cases for the Lipschitz coefficients of the functions associated with this part. It is assumed that the linear part of the equation considered admits an exponential dichotomy. Finally, an application is given on the very interesting model of Markus and Yamabe.

References

1. Elhadi Ait Dads, Safoua Khelifi and Mohsen Miraoui, *On the integro-differential equations with reflection*, Mathematical Methods in the Applied Sciences, (2020).

Decidability in Group Cellular Automata

Jarkko Kari

Many undecidable questions concerning cellular automata are known to be decidable when the cellular automaton has a suitable algebraic structure. Typical situations include linear cellular automata where the states come from a finite field or a finite commutative ring, and so called additive cellular automata in the case the states come from a finite commutative group and the cellular automaton is a group homomorphism. In this talk we generalize the setup and consider so-called group cellular automata whose state set is any (possibly non-commutative) finite group and the cellular automaton is a group homomorphism. The configuration space may be any group shift - a subshift that is a subgroup of the full shift - and still many properties are decidable in any dimension of the cellular space. Our decidability proofs are based on algorithms to manipulate group shifts, and on viewing the set of space-time diagrams of group cellular automata as multidimensional group shifts. The trace shift and the limit set of the cellular automaton are lower dimensional projections of the space-time diagrams and they can be effectively constructed. This view provides algorithms to decide injectivity, surjectivity, equicontinuity, sensitivity and nilpotency of the cellular automaton. Non-transitivity is semi-decidable. We also easily establish that injectivity always implies surjectivity, that transitivity implies mixingness, that non-sensitivity implies equicontinuity, and that jointly periodic points are dense in the limit set. The talk is based on a joint work with Pierre Baur.

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1. Pierre Baur, Jarkko Kari. Decidability in Group Shifts and Group Cellular Automata. In: Proceedings of MFCS 2020. Leibniz International Proceedings in Informatics (LIPIcs) 170, 12:1-12:13, 2020.

Topological rank of cantor factors of cantor minimal systems

Maryam Hosseini and Nasser Golestani
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Abstract: A Cantor minimal system is of finite topological rank if it has a Bratteli-Vershik representation whose number of vertices per level is uniformly bounded. We prove that if the topological rank of a minimal dynamical system on a Cantor set is finite then all its minimal Cantor factors have finite topological rank as well. This gives an affirmative answer to a question posed by Donoso, Durand, Maass, and Petite in full generality. As a consequence, we obtain the dichotomy of Downarowicz and Maass for Cantor factors of finite rank Cantor minimal systems: they are either odometers or subshifts.

Discrete Pseudo Almost Periodic Solutions for Some Difference Equations

Elhadi Ait Dads, Khalil Ezzinbi and Lahcen Lhachimi
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Abstract: In this work, we study the existence and uniqueness of pseudo al- most periodic solutions for some difference equations. Firstly, we investigate the spectrum of the shift operator on the space of pseudo almost periodic sequences to show the main results of this work. For the illustration, some applications are provided for a second order differential equation with piecewise constant arguments.
This work is organized as follows. In section 2, we consider geometrical properties of the shift operator in general case and, we deal with the properties of shift operator the spaces of almost periodic and on ergodic sequences. In section 3, we consider the existence and uniqueness solutions of some difference equations using polynomial functions. In the last section, we deal with the application of the previous results to some second order differential equation with a piecewise constant argument.

Keywords: Difference equations, pseudo almost periodic sequences, schift operator.

Applications of the Fixed Point Theory

Svetlin G. Georgiev

Many problems in science lead to nonlinear equations $Tx + Fx = x$ posed in some closed convex subset of a Banach space. In particular, ordinary, fractional, partial differential equations and integral equations can be formulated like these abstract equations. It is the reason for which it becomes desirable to develop fixed point theorems for such equations. When T is compact and F is a contraction there are many classical tools to deal with such problems. The main aim of this talk is to give some recent results for existence of fixed points for some operators that are of the form $T + F$, where T is an expansive operator and F is a k-set contraction and the talk offers an overview of recent developments of fixed point theorems. They are given applications for existence of solutions for IVPs and BVPs for ODEs, PDEs, impulsive dynamic equations, fuzzy dynamic equations and dynamic inclusions.

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Lambert W function in the stability and bifurcation analysis of homographic Ricker maps

J. Leonel Rocha [(a)] and Abdel-Kaddous Taha [(b)]

(a) CEAUL. ADM, ISEL-Engineering Superior Institute of Lisbon,
Polytechnic Institute of Lisbon, Portugal.

(b) INSA, Federal University of Toulouse Midi-Pyrénées, Toulouse, France.

Abstract: Dynamical systems of the type homographic Ricker maps are considered, which are particular cases of a new extended-Ricker population model: a discrete-time population model whose dynamics of the population x_n, after n generations, with $n \in N$, can be defined by the difference equation $x_{n+1} = b(x_n)x_n^{\gamma-1}s(x_n)$, and written in the following form

$$x_{n+1} = r\frac{x_n^{\gamma}}{\beta + x_n}e^{-\delta x_n}$$

where is the cooperation or Allees effect parameter. The per-capita birth or growth function $b(x_n) = \frac{cx_n}{\beta+x_n}$ is a Hollings type II functional form or rectangular hyperbola, where $c > 0$ measures the maximal reproduction or growth rate and the ratio c/β measures the relative growth rate as the population size is smaller. The survival function for generation n or the intraspecific competition is given by $s(x_n) = e^{\mu - x_n}$, where $\mu > 0$ is the density-independent death rate, $\delta > 0$ is the carrying capacity parameter, with $r = ce^{\mu}$, γ and β real parameters.

The purpose of this talk is to investigate the nonlinear dynamics properties of the homographic Ricker maps, denoted by $f(x; r, \delta, \beta)$, for some particular cases of the parameter. Then we study the fixed points of these homographic maps as analytical solutions of Lambert W functions. Using general properties of Lambert W functions, we establish conditions for the existence, nature and stability of the non-zero fixed points. Throughout this work, we will show how the use of LambertW functions are useful to formalize analytical results and to represent bifurcation curves. Fold and flip bifurcation structures of the homographic Ricker maps are investigated, in which there are flip codimension-2 bifurcation points and cusp points, while some parameters evolve. Some communication areas and big bang bifurcation curves are also detected, see Fig.1. Several numerical simulations illustrate the theoretical results established.

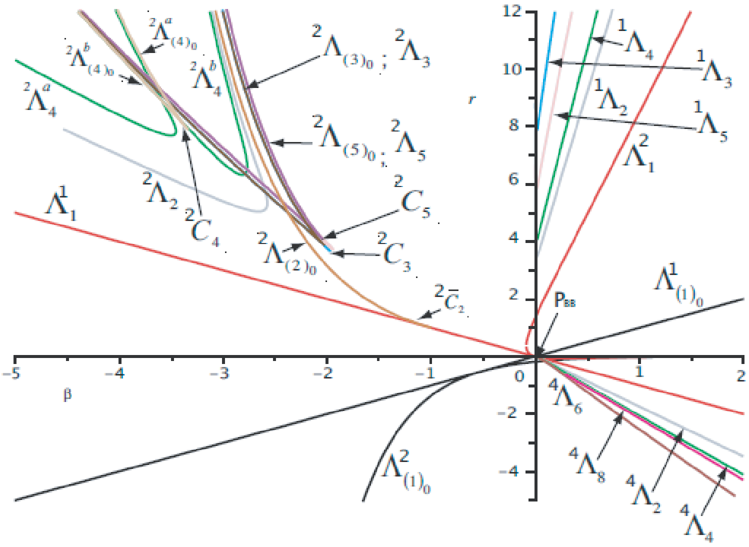

Figure 1: Bifurcation curves of the homographic Ricker map $f(x;r,\delta,\beta)$ in the $\Delta_{\beta,r}$ parameter plane at $\delta=2$: ${}^{k}\Lambda^{j}_{(n)_0}$ are the fold bifurcation curves of the cycles of order $n=1,2,3,4,5$, where k denotes the quadrant and j is the differentiation between curves of the same cycle; ${}^{k}\Lambda^{j}_{n}$ are the flip bifurcation curves of the cycles of order $n=1,2,3,4,5,6,8$; ${}^{2}\overline{C}_2=\Lambda^1_1\cap{}^{2}\Lambda_{(2)_0}$ is a flip codimension–2 bifurcation point; ${}^{2}C_3$, ${}^{2}C_4$ and ${}^{2}C_5$ are the cusp points related to the cycles of order $n=3,4,5$, respectively; $P_{BB}\in\tilde{\Lambda}$ is a big bang bifurcation point.

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Discrete dynamical systems

Participants :

1. Yahiaoui Yaniss. Bejaia University, Algeria.
2. Tarek Sellami. Sfax University, Tunisia.
3. Hacene Gharout. Bejaia University, Algeria.
4. Saliha Djenaoui. Guelma University, Algeria.
5. Smaïl Kaouache. Centre universitaire de Mila, Algeria.
6. Amira Ayari. Annaba University, Algeria.
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11. Zaamoune Faiza. Biskra University, Algeria.
12. Bouharket Benaissa. Tiaret University, Algeria.
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14. Nouar Chorfi. Tebessa University, Algeria.
15. Merzoug Ibtissem. Annaba University, Algeria.
16. Mouna Yahiaoui. Bejaia University, Algeria.
17. Nouressadat TOUAFEK. Jijel University, Algeria.
18. Tinhinane Meziani. University of Bejaia, Algeria.

Sur les bifurcations d'un certain système dynamique discret

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Résumé : L'objectif de ce travail est d'étudier la dynamique et les bifurcations d'un système dynamique discret particulier. La bifurcation de Neimark-Sacker est analysée algébriquement et illustrée par des simulations numériques. De plus, des bifurcations globales sont observées par simulation. Il est à noter que l'étude de la succession des bifurcations permet de comprendre les mécanismes qui conduisent à l'apparition du chaos. En effet, plusieurs attracteurs chaotiques ont été observés dans le plan de phase pour certaines valeurs particulières des paramètres.

Mots clés: Systèmes dynamiques discrets; bifurcation de Neimark-Sacker; bifurcations globales; bassins d'attraction; attracteurs chaotiques.

Références

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Commun dynamics of Rauzy fractals with the same incidence matrix

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Abstract: The matrix of a substitution is not sufficient to completely determine the dynamics associated with it, even in the simplest cases since there are many words with the same abelianization.
In this talk we study the common points of the canonical broken lines associated with two different irreducible Pisot unimodular substitutions σ_1 and σ_2 having the same incidence matrix. We prove that if 0 is an inner point to the Rauzy fractal associated with the substitution σ_1, and σ_1 verifies the Pisot conjecture then these common points can be generated with a substitution on an alphabet of so-called balanced pairs, and we obtain in this way the intersection of the interior of two Rauzy fractals.

Evolution of a three-dimensional endomorphism towards hyper chaos

Hacene Gharout[(1)], Nourredine Akroune[(1)] and Abdel-Kaddous Taha[(2)]

[(1)] Laboratoire des Mathématiques Appliquées, Faculté des Sciences Exactes, Université de Bejaia, 06000 Bejaia, Algeria.

[(2)] INSA, University of Toulouse, 135 Avenue de Rangueil, INSA de Toulouse, France.

Abstract: The aim of this work is to study the chaotic dynamics of a non-invertible and non-linear three-dimensional transformation, called endomorphism, through contact bifurcations with a new type of critical manifold. The critical manifolds observed in the case of dimension three are different from the critical points and critical lines known in dimensions one and two.

keywords: critical spaces, closed invariant curve, contact bifurcation, weakly chaos, chaos.

We will focus on the endomorphism T, defined by:

$$T \begin{cases} x_{n+1} = y_n, \\ y_{n+1} = z_n, \\ z_{n+1} = x_n^2 + a y_n (x_n + z_n) + b. \end{cases}$$

where a and b are two real parameters.

The equation of the critical manifold EC_{-1} of T satisfies $\mid J(x,y,z) \mid = 0$, where $J(x,y,z)$ is the Jacobian of T at the point (x,y,z): $EC_{-1} = \{(-\frac{a}{2}y, y, z), y, z \in \mathbb{R}\}$.
$EC_0 = T(EC_{-1}) = \{(y, z, \frac{1}{4}a^2y^2 + ay(-\frac{1}{2}ay + z) + b),\ y, z \in \mathbb{R}\}$.
The critical spaces of order $n+1$, are defined by $EC_{n+1} = T(EC_n)$ for all $n \geq 0$.
An equivalent way for an order n, $n \geq 0$, the critical varieties are defined by
$EC_n = T^{n+1}(EC_{-1})$ (see figure 1).

T admits two chaotic dynamics, by varying one of the parameters a and b:
1. For b fixed and a varies, T has two dynamics for the variations of a, positively and negatively, which evolve towards chaotic attractors.
2. For a fixed and b varies, we have the coexistence of attractors which evolve towards chaotic attractors (see figure 2), then towards the same hyper chaotic attractor, having a Lyapunov dimension equal to 3 and three positive Lyapunov exponents.

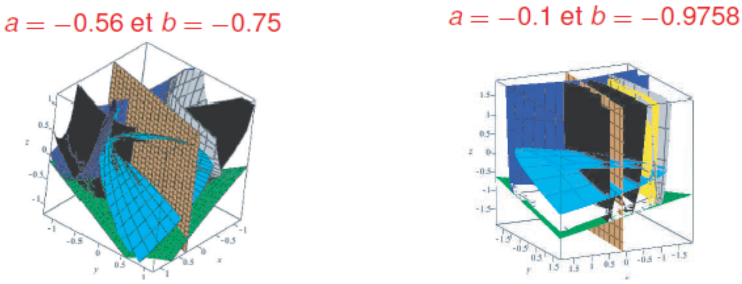

Figure 1: Critical sapces: EC_{-1} in brown, EC in green, EC_1 in blue, EC_2 in gray, EC_3 in cyan, EC_4 in orange, EC_5 in black, EC_6 in white and EC_8 in yellow.

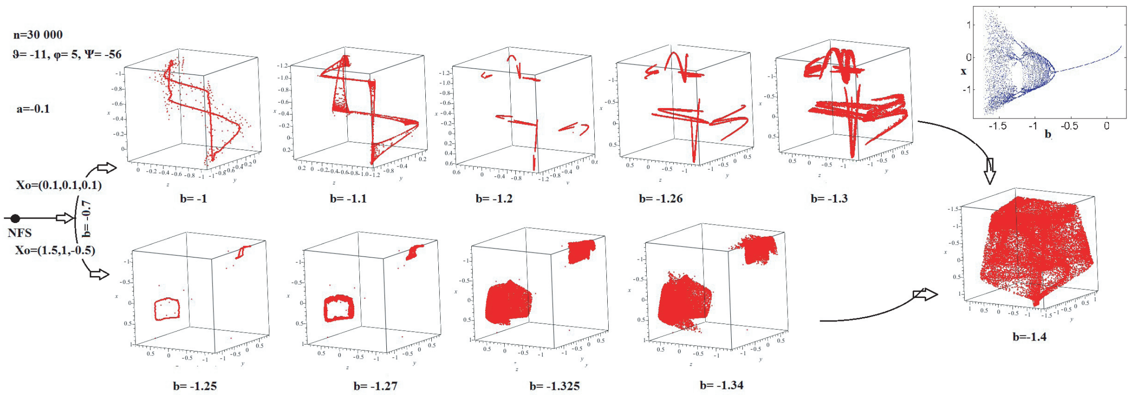

Figure 2: $b = -0.1$ and b is variable.

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Overview of the generic limit set

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Abstract: In topological dynamics, the generic limit set is the smallest closed subset which has a comeager realm of attraction. We study some of its topological properties, and the links with equicontinuity and sensitivity. We emphasize the case of cellular automata, for which the generic limit set is included in all subshift attractors, and discuss directional dynamics. Its main properties are:

- The generic limit set of a nonwandering system (in particular, of a surjective CA) is full.
- The generic limit set of a semi-nonwandering system (in particular, of an oblique CA) is its limit set.
- The generic limit set of an almost equicontinuous system is exactly the closure of the asymptotic set of its set of equicontinuity points.
- The generic limit set of an equicontinuous dynamical system is its limit set. Moreover, if one has an equicontinuous cellular automaton such that its generic limit set is finite, then it is nilpotent.
- The generic limit set of a cellular automaton which is almost equicontinuous in two directions of opposite sign is finite; it is the periodic orbit of a monochrome configuration.
- The generic limit set of a sensitive system is infinite.

Keywords: Cellular automaton, topological system, limit set, basin of attraction

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Chaos and mixed combination synchronization of three identical fractional hyperchaotic systems with different fractional-order

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Abstract. In this work, we introduce a new approach to investigate mixed combination synchronization (MCS) of three identical fractional hyperchaotic systems with different order. Specifically, this kind of synchronization is coexistence between complete combination synchronization, combination anti-synchronization, projective combination synchronization and modified projective combination synchronization. With the help of the stability theory of fractional-order systems, an active controller is designed to assure that the MCS is achieved. Finally, we take the fractional Lorenz system as an example to show the effectiveness of the proposed synchronization scheme.
Keywords. Combination synchronization, Caputo differential operator, Active control, Fractional-order chaotic systems.

1. Problem formulation of the MCS

In this section, we introduce the concept of the MCS of three identical fractional hyperchaotic dystems with different fractional-order. The model can be given as follow:

$$D^{\alpha}x = f(x), \tag{1}$$

$$D^{\alpha}y = g(y), \tag{2}$$

$$D^{\alpha}z = h(z) + u, \tag{3}$$

where D^{α} is the Caputo differential operator ($0 < \alpha \leq 1$), $x = (x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4$, $y = (y_1, y_2, y_3, y_4)^T \in \mathbb{R}^4$ are the state variables of two drive systems, $z = (z_1, z_2, z_3, z_4)^T \in \mathbb{R}^4$ is the state variable of the response system, $f, g, h : \mathbb{R}^4 \to \mathbb{R}^4$ are the continuous vector-valued functions and $u = (u_1, u_2, u_3, u_4)^T \in \mathbb{R}^4$ is the controller vector which will be designed.

The definition of the proposed mixed combination synchronization is given as follows.

Definition: The two drive systems (1)-(2) and the response system (3) are said to achieve the MCS between complete combination synchronization, combination anti-synchronization, projective combination synchronization and modified projective combination synchronization if there exist controller u and two constant matrices $B = diag(1,\ 1,\ 1,\ \sigma)$ and $C = diag(1, -1, \beta, 1)$ such that the synchronization error: $e(t) = Bz(t) - C(y(t) + x(t)$, satisfy the condition $\lim_{t\to\infty} ||e(t)|| = 0$, where $||.||$ stands for the matrix norm.

Now, from equations (1), (2) and (3) , we can get the following error system:

$$\begin{aligned} D^{\alpha}e &= BD^{\alpha}z - CD^{\alpha}(x+y) \\ &= Ae + (BA - AB)z - (CA - AC)(x+y) + \\ &+ \ BH(z) - C(F(x) + G(y)) + Bu, \end{aligned}$$

where $A \in \mathbb{R}^{4\times 4}$ is the linear part of the system, F, G and $H : \mathbb{R}^4 \to\in \mathbb{R}^4$ are the nonlinear parts.

To achieve the desired MCS between the above systems, the nonlinear active controller $u = (u_1, u_2, u_3, u_4)^T$ is constructed as:

$$u = B^{-1}((CA - AC)(x+y) - (BA - AB)z + C(F(x) + G(y)) - BH(z) + Me), \qquad (4)$$

where $M \in \mathbb{R}^{4\times 4}$ is a feedback gain matrix to be determined.

So, when we use the controller (4) to control the fractional-order response system (3), the MCS problem of the fractional-order drive systems (1)-(2) and fractional-order response system (3) is changed into the analysis of the asymptotical stability of the following system:

$$D^{\alpha}e = (A + M)e.$$

Then, we have the following result.

Theorem: If the matrix M is selected such that all roots λ_i of the characteristic equation:

$$\det(diag(\lambda^{s\alpha_1},\ \lambda^{s\alpha_2},\ \lambda^{s\alpha_3},\ \lambda^{s\alpha_4}) - (A + M)) = 0,$$

satisfy $|\arg(\lambda_i)| > \dfrac{\pi}{2s}, i = 1, 2, 3, 4$, where s is the least common multiple of the denominators of λ_i, then the two drive systems (1)-(2) and response system (3) can be synchronized in the sens of MCS under the controller (4).

2. Conclusion

In this work, we have presented a new approach to study the problem of mixed combination synchronization of three identical fractional hyperchaotic systems with different fractional-order. In particular, this work has shown that complete combination synchronization, combination anti-synchronization, projective combination synchronization and modified projective combination synchronization coexist when synchronizing two master system with one response system. The numerical example reported through the paper has clearly highlighted the capability of the proposed approach.

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Sufficient conditions for exponential stability of some non linear perturbed system on time scales

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Abstract: This paper focuses on the problem of exponential stability of certain classes of dynamic perturbed systems on time scales using time scale versions of some Gronwall type inequalities.We prove under certain conditions on the nonlinear perturbations that the resulting perturbed nonlinear initial value problem still acquir exponential stable, if the associated time-varying linear system has already owned this property. Furthermore, one example is given to illustrate the applicability of the obtained results.

Keywords: Dynamic equation, time scale, Gronwall inequality, Exponential stability.

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger [5] in his Ph. D. thesis in 1988 in order to unify continuous and discrete analysis. A great deal of work has been done since 1988, unifying the theory of differential equations and the theory of difference equations by establishing the corresponding results in time scale setting. A time scale $\mathbf{T}$ is an arbitrary nonempty closed subset of the set of real numbers $\mathbb{R}$. During the last decades, time scale methods have rapidly been developed, and have received a lot of attention by several authors, not only to unify continuous and discrete processes, but also help reveal diversities in the corresponding results.The analysis of nonlinear perturbations of linear systems is not only important for its own sake but also has a broad range of applications.

One of the analytic methods of the perturbation theory was referred to integral inequalities to quest some type of stability. Latterly, there have been several papers $[1, 4]$, studying various types of stability of dynamical time scale systems.

In this work, we investigate uniform exponential stability for nonlinear perturbed systems on time scales by using the Gronwall- Bellman-Bihari type integral inequality.

1.1. Time scale calculus

In what follows, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_+ = [0, \infty)$ is the given subset of $\mathbb{R}$ and $\mathbf{T}$ is an arbitrary time scale. The forward and backward jump operators $\sigma, \rho : \mathbf{T} \to \mathbf{T}$ are defined by $\sigma(t) := \inf \{s \in \mathbf{T} : s > t\}$, $\rho(t) =: \sup \{s \in \mathbf{T} : s < t\}$. Also we define the interval $[a,b]$ means the set $\{t \in \mathbf{T} : a \leq t \leq b\}$ for the points $a < b$ in $\mathbf{T}$. If $b = +\infty$, we denote $\mathbf{T}_a^+ = [a, +\infty[$ .for reader's convenience, as they are detailed in the books of M. Bohner and A. Peterson [2, 3].

1.2. Stability definitions

For our purpose, we will assume that the time scale $\mathbf{T}$ is unbounded above, i.e., $\sup \mathbf{T} = +\infty$. Let $t_0 \in \mathbf{T}$ and $t \in \mathbf{T}_{t_0}^+$. Let us consider time scale dynamic equations of the form

$$\begin{aligned} x^{\Delta}(t) &= f(t, x(t)), \\ x(t_0) &= x_0. \end{aligned} \tag{1}$$

where $x : \mathbf{T}_{t_0}^+ \to \mathrm{I\!R}^n$ is the state vector and $f : \mathbf{T}_{t_0}^+ \times \mathrm{I\!R}^n \to \mathrm{I\!R}^n$ is a rd-continuous vector-valued function. It is assumed that the conditions for the existence of a unique solution of system (1) are satisfied. For the existence, uniqueness and extensibility of its solutions, one can refer to [2] .Designate any solution of (1) with the initial state (t_0, x_0) by $x(t) = x(t, t_{0,} x_0)$.The Euclidean norm of an $n \times 1$ vector x(t) is defined to be a real-valued function of t and is denoted by $\|x(t)\| = \sqrt{x(t)^T x(t)}$.

Definition:

The system of dynamic equations (1) is said to be uniformly exponentially stable if there exist constants $\gamma \geq 1$ (independent of t_0) , $\lambda > 0$ $(-\lambda \in \Re^+)$ such that

$$\|x(t)\| \leq \gamma \|x_0\| e_{-\lambda}(t, t_0).$$

Now, we give the following characterization in terms of the transition matrix for system (1).

2. Main Results

In this study, we consider, a particular class of systems (1), i.e the system

$$\begin{aligned} x^{\Delta}(t) &= A(t)x + F(t, x(t)), \\ x(t_0) &= x_0. \end{aligned} \tag{2}$$

where $x_0, x \in \mathrm{I\!R}^n$, $F(t, 0) = 0$, $t_0 \in \mathbf{T}$, and $F : \mathbf{T} \times \mathrm{I\!R}^n \to \mathrm{I\!R}^n$ is an rd-continuous function. f represents the disturbance of the time-varying linear system :

$$\begin{aligned} x^{\Delta}(t) &= A(t)x, \\ x(t_0) &= x_0, x_0 \neq 0. \end{aligned} \tag{3}$$

We prove under certain conditions on the nonlinear perturbations that if system (3) is uniformly exponentially stable then system (2) has the same propriety.

3. Numerical examples

Let $\mathbf{T}$ be a mixed continuous-discrete time scale and $t_0 = 0$.The discrete part has non-uniform step size. The graininess function is bounded as follows: $\forall t \in \mathbf{T}_0^+$

$$0 \leq \mu(t) < \mu_{\max} = \frac{1}{2}.$$

Consider the following time-varying system:

$$\begin{array}{ll} x_1^{\Delta}(t) & = -x_1(t) + \frac{1}{2}\ln\left(\frac{1}{(t+1)(\sigma(t)+1)}\left|x_1(t)\right| + \frac{k(t)|x_2(t)|}{\sqrt{x_1^2(t)+x_2^2(t)+1}} + 1\right), \\ x_2^{\Delta}(t) & = -x_2(t) + \frac{\sqrt{3}}{2}\ln\left(\frac{1}{(t+1)(\sigma(t)+1)}\left|x_2(t)\right| + \frac{k(t)|x_1(t)|}{\sqrt{x_1^2(t)+x_2^2(t)+1}} + 1\right), \\ x(0) & = (x_{1,0}, x_{2,0}), \end{array} \tag{5}$$

where $x = (x_1, x_2)^T \in \mathbb{R}^2$, $k \in C_{rd}(\mathbf{T}, \mathbb{R}_+)$ and $k(t) = \frac{t+\sigma(t)+2}{(t+1)^2(\sigma(t)+1)^2} e_{-\lambda}(\sigma(t), 0)$.

We prove that the above system is uniformly exponentially stable subject to some sufficient conditions.

Conclusion

This paper has been concerned with the problem of exponential stability for nonlinear system .Sufficient conditions for exponential stability of a class of dynamic systems on arbitrary time scales are obtained using integral inequalities approach. On the mentioned topics, new theorems are proven. The obtained results include and improve some results in the literature.Moreover, two examples are given to illustrate the applicability of the main result.

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ON RECURRENCE IN DENDRITE FLOWS

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Abstract: Let G be a finitely generated group acting by homeomorphisms on a dendrite X. We show that (G, X) is a pointwise recurrent flow if one of the following two statements holds: (G, X) is a minimal and proximal flow; and cutpoints of X is periodic and every non periodic endpoint belongs to a G-subodometer.

Keywords: Dendrite, group action, minimal set, recurrent, proximal.

Introduction

Given a topological group G and a compact topological space X, in the sequel, a flow (G, X), stands for a continuous action of G on X. One of the main problems concerning studying a flow or a transformation group (G, X) is the relationships between the following dynamical notions: (1) pointwise recurrence, (2) almost periodicity, (3) the orbit closure relation is closed, and (4) equicontinuity in the setting of finitely generated group action on a compact metric space. Several authors have been interested in studying the relations between the above notions. In [1, 4], it is shown that the above proprieties are equivalent for a finitely generated group G on either a compact zero-dimensional space or a finite graph space X. Recently, [6], Marzougui and Naghmouchi, proved the equivalence between (2) and (3) in the sitting of local dendrite flow, where G is an arbitrary group.

There has been a particular attention for the study of groups acting on (local)dendrites [9, 8,5]. The interest in studying transformation groups on these spaces is due first to dendrites appear as Julia sets in complex dynamics [2] and second to the study of three-dimensional hyperbolic geometry [8].

1. Flows

A triple (G, X, π) consisting of a topological group G, a compact metric space X and a continuous action $\pi : G \times X \to X$ of G on X is called a *flow* on X ((G, X) for short). For any $x \in X$ the subset $Gx = \{gx : g \in G\}$ is called the *orbit of* x. A point x of X is *periodic under* G if Gx is finite.

A flow (G, X) is minimal if all orbits are dense in X. A point $x \in X$ is almost periodic if for every neighborhood U of X there exists a syndetic subset S of G such that $Sx \subset U$. x is almost periodic if and only if $\overline{Gx}$ is minimal. A flow (G, X) is pointwise almost periodic if every point $x \in X$ is almost periodic.
Two points x and y in X are said to be *proximal* with respect to the action of G if there exists a sequence $\{g_k\}$ in G such that the sequences $\{g_k x\}$ and $\{g_k y\}$ converge to one and the same point. (G, X) is a *proximal* flow if any two points in X are proximal.
The definition of recurrence given in [1]. Let G be a finitely generated group and Let $\Gamma = \{f_1, \dots, f_p\}$ be a finite set of generators. Denote by B_r the set of elements of G of length $\leq r$. For $g \in G$ let $K(g) = B_{|g|-1}.g$ where $|g|$ is the length of g. A subset C of G is a *cone* if there exists a sequence $g_n \in G$ with $|g_n| \to +\infty$ and $C = \lim_{n\to\infty} K(g_n)$. By [1, Proposition 1.5], in a cone C there exists a sequence (c_n) such that $B_n.c_n \subset C$ and for each $g \in G$, $gc_n \in C$ for some n.
Definition: [1] Let (G, X) be a flow where X is a compact metric space and let C be a subset of G such that $e \notin C$. We say that a point $x \in X$ is C-recurrent, if for every neighborhood U of x, $Cx \cap U \neq \emptyset$. We say that x is recurrent, if it is C-recurrent for every cone C.
Let $R(G)$ be the set of recurrent points. A flow (G, X) is pointwise recurrent if $R(G) = X$.
2. Dendrites
A compact connected metric space is called a continuum. An arc is any space homeomorphic to the compact interval $[0, 1]$. A topological space is arcwise connected if any two of its points can be joined by an arc. *A dendrite* D is a locally connected continuum, which contains no simple closed curve (homeomorphic to $\S^1$). Recall that any two distinct points x and y of a dendrite D can be joined by a unique arc with endpoints x and y, denote this arc by $[x, y]$. We put $[x, y) = [x, y] \setminus \{y\}$, $(x, y] = [x, y] \setminus \{x\}$ and $(x, y) = [x, y] \setminus \{x, y\}$. Every sub-continuum of a dendrite is a dendrite. In addition, every dendrite is hereditarily locally connected and every connected subset of a dendrite D is arcwise connected [7].
Main result
Theorem 1. Let G be a finitely generated group acting by homeomorphisms on a dendrite X. We show that (G, X) is a pointwise recurrent flow if one of the following two statements holds:

1. (G, X) is a minimal and proximal flow;

2. Every cutpoints of X is periodic and every non periodic endpoint is regularly recurrent.

Theorem2. Let G be a finitely generated group acting by homeomorphisms on a dendrite X with countable set of endpoints. (G, X) is a pointwise recurrent flow if (G, X) is pointwise periodic.

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Periodicity and factors of endmorphisms of the shift

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Abstract: Endomorphisms of the shift are maps that commutes with the shift. They are characterized by the existence of local function which determine by local behavior the image of an element of the configurations space. Dynamical behavior of CA is studied mainly in the context of discrete dynamical systems by equipping the space of configurations with the product topology which make it homeomorphic to the Cantor space.
We want to characterize equicontinuous factors of endomorphisms of the shift. We show if there is an equicontinuous factor then there is also an equicontinuous endomorphism of the shift as a factor.
We show also that if the endomorphism of the shift has equicontinuity points without being equicontinuous there is an infinity of equicontinuous factors up to conjugacy.
Keywords: Bernoulli shift, endomorphism of the shift, equicontinuous factor.

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Algebraic and non-algebraic limit cycles of a family of planar differential systems

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Abstract: In this work we give an explicit expression of invariant algebraic curves, then we prove that these systems are integrable and we introduce an explicit expression of a first integral of a multi-parameter planar polynomial differential systems of degree nine of the form

$$\begin{cases} x' = \dfrac{dx}{dt} = x + P_5(x,y) + xR_8(x,y), \\ y' = \dfrac{dy}{dt} = y + Q_5(x,y) + yR_8(x,y), \end{cases} \tag{1}$$

where

$$P_5(x,y) = -(a+2)\,x^5 + (4+4b)\,x^4y - (2a+4)\,x^3y^2 + (8+4b)\,x^2y^3 - (a+2)\,xy^4 + 4y^5,$$

$$Q_5(x,y) = -4x^5 - (a+2)\,x^4y + (4b-8)\,x^3y^2 - (2a+4)\,x^2y^3 + (4b-4)\,xy^4 - (a+2)\,y^5,$$

and

$$\begin{aligned} R_8(x,y) \;&=\; (a+1)\,x^8 - 4bx^7y + (4a+4)\,x^6y^2 - 12bx^5y^3 + (6a+6)\,x^4y^4 - 12bx^3y^5 \\ &\quad + (4a+4)\,x^2y^6 - 4bxy^7 + (a+1)\,y^8, \end{aligned}$$

in which a, b are real constants.
Moreover, we determine sufficient conditions for a polynomial differential system to possess two limit cycles : one of them is algebraic and the other one is shown to be non-algebraic, explicitly given. Concrete examples exhibiting the applicability of our result are introduced.

Main result

Our main result is contained in the following theorem.

Theorem

Consider a multi-parameter planar polynomial differential systems (1), then the following statements hold.

1) The origin of coordinates $O\,(0,0)$ is the unique critical point at finite distance.

2) The curve $U(x,y) = x^4 + y^4 + 2x^2y^2 - 1$, is an invariant algebraic curve of system (1) with cofactor $K(x,y) = (-4)\left(x^2+y^2\right)^2 \Big((-a-1)\left(x^2+y^2\right)^2 + 4bxy\left(x^2+y^2\right) + 1 \Big)$.
3) The system (1) has the first integral

$$H(x,y) = \frac{\left(x^2+y^2\right)^2 + \left(1 - \left(x^2+y^2\right)^2\right) \exp\left(a \arctan \frac{y}{x} + b \cos\left(2 \arctan \frac{y}{x}\right)\right) f(\arctan \frac{y}{x})}{\left(\left(x^2+y^2\right)^2 - 1\right) \exp\left(a \arctan \frac{y}{x} + b \cos\left(2 \arctan \frac{y}{x}\right)\right)},$$

where $f(\arctan \frac{y}{x}) = \int_0^{\arctan \frac{y}{x}} \exp(-as - b\cos 2s) ds$.
4) The system (1) has an explicit limit cycle, given in Cartesian coordinates by (Γ_1) : $x^4 + y^4 + 2x^2y^2 - 1 = 0$.
5) If $a > 0$ and $b \in \mathbb{R} - \{0\}$, then system (1) has non-algebraic limit cycle (Γ_2), explicitly given in polar coordinates (r, θ), by the equation

$$r(\theta, r_*) = \left(\frac{\exp(a\theta + b\cos 2\theta) \left(\frac{e^{2\pi a}}{1 - e^{2\pi a}} f(2\pi) + f(\theta) \right)}{-1 + \exp(a\theta + b\cos 2\theta) \left(\frac{e^{2\pi a}}{1 - e^{2\pi a}} f(2\pi) + f(\theta) \right)} \right)^{\frac{1}{4}},$$

where

$$f(\theta) = \int_0^{\theta} \exp(-as - b\cos 2s) ds \ .$$

Moreover, the non-algebraic limit cycle (Γ_2) lies inside the algebraic limit cycle (Γ_1).
Keywords: Limit cycle; Riccati equation; invariant algebraic curve; first Integral.

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A class of planar differential systems with explicit expression for two limit cycles

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Abstract: The existence of limit cycles is interesting and very important in applications. It is a key to understand the dynamic of polynomial differential systems. The aim of this paper is to investigate a class of a multi-parameter planar polynomial differential systems. Under some suitable conditions, the existence of two limit cycles, these limit cycles are explicitly given. Some examples are presented in order to illustrate the applicability of our results.

Keywords: limit cycle; Riccati equation; invariant algebraic curve; first integral.

Introduction

One of the main problems in the qualitative theory of differential equations is the study of the limit cycles of planar differential systems and specially of the planar polynomial differential systems of the form

$$\begin{cases} x' = \dfrac{dx}{dt} = P(x, y), \\ \\ y' = \dfrac{dy}{dt} = Q(x, y), \end{cases} \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials in the variables x and y.

In this paper we give an explicit expression of invariant algebraic curves, then we prove that these systems are integrable and we introduce an explicit expression of a first integral of a multi-parameter planar polynomial differential system of degree nine of the form

$$\begin{cases} x' = \dfrac{dx}{dt} = xS_4(x, y) + P_7(x, y) + xR_8(x, y), \\ \\ y' = \dfrac{dy}{dt} = yS_4(x, y) + Q_7(x, y) + yR_8(x, y), \end{cases} \tag{2}$$

where

$P_7(x,y) = \frac{1}{3}\left(x^2+y^2\right)^2\left(\left(2a-b\right)x^3 + \left(15d-6c\right)x^2y + \left(2b-a\right)xy^2 + \left(6d-3c\right)y^3\right),$

$Q_7(x,y) = -\frac{1}{3}\left(x^2+y^2\right)^2\left(\left(6d-3c\right)x^3 + \left(b-2a\right)x^2y - 3dxy^2 + \left(a-2b\right)y^3\right),$

$S_4(x,y) = \alpha x^4 + \lambda x^3y + \delta x^2y^2 + \lambda xy^3 + \eta y^4$ and

$R_8(x,y) = -\frac{1}{3}\left(x^2+y^2\right)^2\left(\left(3\alpha+2a-b\right)x^4 + \left(3\lambda-3c+9d\right)x^3y + \left(3\lambda-3c+9d\right)xy^3 + \left(a+b+3\delta\right)x^2y^2 + \left(2b-a+3\eta\right)y^4\right),$

in which a, b, c, d, α, λ, δ, η are real constants.

Moreover, we determine sufficient conditions for a polynomial differential system to possess two limit cycles, explicitly given. Concrete examples exhibiting the applicability of our result are introduced.

We define the trigonometric functions

$F(\theta) = \frac{1}{8}\left(3\alpha+\delta+3\eta\right) + \frac{1}{2}\lambda\sin 2\theta + \frac{1}{2}\left(\alpha-\eta\right)\cos 2\theta + \frac{1}{8}\left(\alpha-\delta+\eta\right)\cos 4\theta,$

$G(\theta) = \frac{1}{6}\left(a+b\right) + \frac{1}{2}\left(a-b\right)\cos 2\theta + \frac{1}{2}\left(3d-c\right)\sin 2\theta,$

$K(\theta) = -\frac{1}{6}a - \frac{1}{6}b - \frac{3}{8}\alpha - \frac{1}{8}\delta - \frac{3}{8}\eta + \frac{1}{2}\left(c-3d-\lambda\right)\sin 2\theta + \frac{1}{8}\left(\delta-\alpha-\eta\right)\cos 4\theta + \frac{1}{2}\left(b-a-\alpha+\eta\right)\cos 2\theta$, $M(\theta) = \int_0^\theta \left(\frac{2K(t)}{2d-c}\exp\left(\int_0^t\left(\frac{2G(w)+4K(w)}{c-2d}\right)dw\right)\right)dt$ and $N(\theta) = \exp\left(\int_0^\theta\left(\frac{2G(w)+4K(w)}{c-2d}\right)dw\right)$.

Main result

Our main result is contained in the following Theorem.

Theorem:

Consider a multi-parameter planar polynomial differential system (2), then the following statements hold.

1) If $2d-c \neq 0$, then the origin of coordinates $O(0,0)$ is the unique critical point at finite distance.

2) The curve $U(x,y) = x^2+y^2-1$, is an invariant algebraic curve of system (2) with cofactor

$$\begin{aligned} K(x,y) \;=\; & -\frac{2}{3}\left(x^2+y^2\right)\left(\left(2a-b+3\alpha\right)x^6 + 3\alpha x^4 + 3\eta y^4 + \left(9d-3c+3\lambda\right)xy\left(x^2+y^2\right)^2 + \right. \\ & \left. 3x^2y^2\left(\left(a+\alpha+\delta\right)x^2 + \left(b+\delta+\eta\right)y^2 + \delta\right) + 3\lambda xy\left(x^2+y^2\right) + \left(2b-a+3\eta\right)y^6\right). \end{aligned}$$

3) The system (2) has the first integral

$$H(x,y)=\frac{N\left(\arctan\frac{y}{x}\right)+\left(1-x^2-y^2\right)M\left(\arctan\frac{y}{x}\right)}{x^2+y^2-1}.$$

4) The system (2) has an explicit limit cycle, given in Cartesian coordinates by (Γ_1): $x^2+y^2-1=0$.

5) If

$$\begin{aligned}&\tfrac{2}{3}a+\tfrac{2}{3}b+3\alpha+\delta+3\eta>2\left|c-3d-2\lambda\right|+2\left|b-a-2\alpha+2\eta\right|+\left|\delta-\alpha-\eta\right|,\\&-\tfrac{1}{3}a-\tfrac{1}{3}b-\tfrac{3}{4}\alpha-\tfrac{1}{4}\delta-\tfrac{3}{4}\eta>\left|c-3d-\lambda\right|+\tfrac{1}{4}\left|\delta-\alpha-\eta\right|+\left|b-a-\alpha+\eta\right|,\\&\delta\neq\alpha+\eta \text{ and } c<2d,\end{aligned}$$

then the system (2) has another limit cycle (Γ_2), explicitly given in polar coordinates (r,θ) by

$$r(\theta,r_*)=\sqrt{\frac{\left(N(2\pi)-1\right)\left(N(\theta)+M(\theta)\right)+M(2\pi)}{\left(N(2\pi)-1\right)M(\theta)+M(2\pi)}}.$$

Moreover, the limit cycle (Γ_1) lies inside the limit cycle (Γ_2).

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Discovering Hidden Bifurcation in Chua system Via Transformation

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Abstract: In this paper, we give formally a new method discovery hidden bifurcations in the multispiral Chua attractor. This method is based on the mind idea of the genuine Leonov and Kuznetov technique searching for hidden attractrs but applied in a extremely different way where the numbers of spiral happening is discrete. In this case such hidden bifurcations are controled by a parameter ε, this added which is not be found from the initial problem is totally made to show the real structure of multispiral attractor. We study completely the multispiral Chua attractor, generated via transformation, and a verification numerically our method for unusual and equal the number of spiral from 2 to 6.

Keywords: Chua system, transformation, hidden bifurcations. **Abstract:** In this paper, we give formally a new method discovery hidden bifurcations in the multispiral Chua attractor. This method is based on the mind idea of the genuine Leonov and Kuznetov technique searching for hidden attractrs but applied in a extremely different way where the numbers of spiral happening is discrete. In this case such hidden bifurcations are controled by a parameter ε, this added which is not be found from the initial problem is totally made to show the real structure of multispiral attractor. We study completely the multispiral Chua attractor, generated via transformation, and a verification numerically our method for unusual and equal the number of spiral from 2 to 6.

Keywords: Chua system, transformation, hidden bifurcations.

Introduction

In the framework of the specific theory chaotic systems, the name "Chua attractors " is these days extremely used because it is the asymptotic attractor of settlements of the system of differential equations designing the dynamics of the Chua's circuit. The Chua's system is among the systems that have been studied for the last decades, due to their promising applications in various real-world technologies. Newly a novel notion about the classification of attractors has been presented: periodic to the type of hidden attractors. In $2009-2010$, Leonov Kuznetsov proposed an analytical-numerical method for the location of hidden attractors in dynamic systems [2]. In 2016 Menacer, et al.[1] applied analytical-numerical methods for investigation of hidden attractors in the Chua system via since function, they noticed that have a change in the numbers of scrolls when we change the parameter epsilon in a system, this change they called hidden bifurcations. After that by three years later, Zaamoune et al [3], applied a hidden bifurcation method in other systems (design and analysis of multi scroll chaotic attractors from saturated function series), they discovered symmetries in the hidden bifurcation routes. The novelty that this article introduces is, the study of hidden bifurcation of multi scroll chaotic attractors via parallel transformation with a method based on a homotopy parameter ε whilst conserving the number of scrolls constant [1].

Chua's system with $1-D$ scroll chaotic attractor generated via parallel transformation

Here, to generate $1-D$ n scroll chaotic attractor, we present a Chua's system from parallel transformation as folow :

$$\begin{cases} \dot{x} = a(y - h(x) \\ \dot{y} = x - y + z \\ \dot{z} = -by \end{cases}, \tag{1}$$

where $a = 10$, $b = 16$ and $h(x)$ is the nonlinear function, defined by one of the two following forms:

To obtain an even number n of scrolls according to the formula ($n = 2N + 2$), the function $h(x)$ is given as follow

$$h(x) = kx - pk\left[-sgn(x) + \sum_{i=0}^{N}(x + 2ip) - \sum_{i=0}^{N}(x - 2ip)\right] \tag{1}$$

where p, k are real numbers.

$$sgn\,(x) = \begin{cases} +1 \text{ if } x > 0 \\ -1 \text{ if } \ x < 0 \end{cases}$$

For $a = 10$, $b = 16$, $p = 0.5$, and $k = 0.3$, $6 - scroll$ are generated as the verged chaotic attractor of system (1), with both formulas, respectively (2), as shown in fig.1 and fig.2. a, b, p, k are real numbers. The formula to calculated the number n of scrolls, it's explained in Kehui Sun et al [4] which equalizes ($n = 2N + 2$).

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New proof of Hardy dynamic Integral Inequality on Time Scales

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Abstract: In this paper, we derive new proof of Hardy dynamic integral inequality by using the dynamic Minkowski integral inequality on time scale and we also use an interesting lemma.

Keywords: Minkowski integral inequality, Time scales.

1. Introduction and preliminaries

In 2005 Pavel Rehak [3] presented a Hardy inequality on time scales:

Theorem 1. Let $p > 1$ be a constant, a function f be nonnegative and such that the delta integral $\displaystyle\int_a^x f^p(t)\Delta t$ exists as a finite number. Denote $F(t) := \displaystyle\int_a^x f(t)\Delta t$. Then

$$\int_a^\infty \left(\frac{F^\sigma(x)}{\sigma(x)-a}\right)^p \Delta x \leq \left(\frac{p}{p-1}\right)^p \int_a^\infty f^p(x)\Delta\, x. \tag{1}$$

unless $f \equiv 0$. If, in addition, $\frac{(x)}{x} \to 0$ as $x \to \infty$, then the constant is the best possible.

The aim of this presentation is to obtain a similar result with a new proof by utilizing a variant of the Minkowski inequality on time scales and a new lemma derived from chain rule [1].

From the Theorem 9.1 (page 185) [2], we can drive the following corollary.

Let $0 \leq a < b \leq +\infty$ and $0 \leq c < d \leq +\infty$, we introduce the rectangle R in $\mathbf{T}_1 \times \mathbf{T}_2$, defined by $R = [a,b) \times [c,d) = \{(s,t);\ s \in [a,b)\ and\ t \in [c,d)\}$.

Corollary 1. Let $1 \leq p < \infty$, $\phi :\ R_1 \longrightarrow \mathbb{R}$ be a continuous function on $\mathbf{T}_1 \times \mathbf{T}_2$ and $f(s,t) \in L^p_\Delta([a,b))$ for almost all $t \in [c,d)$. Then

$$\int_a^b \left|\int_c^d \phi(s,t)\Delta_2 t\right|^p \Delta_1 s \leq \left(\int_c^d \left(\int_a^b |\phi(s,t)|^p \Delta_1 s\right)^{\frac{1}{p}} \Delta_2 t\right)^p. \tag{2}$$

hold if the right-hand side is finite.

Now we give the lemma which will be used in the proof of main theorem.

Lemma 1. For $1 \le p < \infty$, we have

$$\int_0^1 \frac{1}{(\sigma(s))^{\frac{1}{p}}} \Delta\, s \le \frac{p}{p-1}. \tag{3}$$

Proof. Let h be a nonnegative and non-decreasing function on $[a,\, b]_{\mathbf{T}}$, applying the chain rule for $-1 < q < 0$, we get

$$\begin{aligned}
(h^{q+1})^{\Delta} &= (q+1)\, h^{\Delta} \int_0^1 (r\, h^{\sigma} \,+ (1-r)h)^q\, dr \\
&\ge (q+1)\, h^{\Delta} \int_0^1 (r\, h^{\sigma} \,+ (1-r)h^{\sigma})^q\, dr \\
&= (q+1)\, h^{\Delta}(h^{\sigma})^q.
\end{aligned}$$

Let $p > 1$ and taking $h(s) = s,\;\; q = -\frac{1}{p}$, we deduce that

$$(\sigma(s))^{-\frac{1}{p}} \le \frac{p}{p-1}(s^{1-\frac{1}{p}})^{\Delta},$$

by integrating the above inequality, we get

$$\int_0^1 (\sigma(s))^{-\frac{1}{p}} \Delta\, s \le \frac{p}{p-1} \int_0^1 (s^{1-\frac{1}{p}})^{\Delta} \Delta\, s = \frac{p}{p-1}.$$

2. Main result

The dynamical Hardy integral inequality

Theorem 2. Let $\mathbf{T}$ be a time scales, $a \in \mathbf{T}$, $p > 1$ and g be non-negative continuous functions on $[a,\, \infty)_{\mathbf{T}}$, let

$$G(x) = \int_a^x g(t)\Delta t.$$

If $\Delta(\sigma(x)) = \Delta\, x$, then

$$\int_a^{\infty} \left(\frac{G^{\sigma}(x)}{\sigma(x) - a}\right)^p \Delta x \le \left(\frac{p}{p-1}\right)^p \int_a^{\infty} g^{\,p}(x)\Delta\, x. \tag{4}$$

Proof. Taking $t = (x-a)\sigma(s) + a$, then

$$G(x) = \int_a^x g(t)\Delta t = (x-a) \int_0^1 g((x-a)\sigma(s) + a)\Delta s,$$

by using Minkowski's inequality (2), we have

$$\begin{aligned}\int_a^\infty \left(\frac{G^\sigma(x)}{\sigma(x)-a}\right)^p \Delta\, x &= \int_a^\infty \left(\int_0^1 g((\sigma(x)-a)\sigma(s)+a)\Delta s\right)^p \Delta\, x \\ &\le \left(\int_0^1 \left[\int_a^\infty g^{\,p}((\sigma(x)-a)\sigma(s)+a)\Delta x\right]^{\frac{1}{p}} \Delta\, s\right)^p\end{aligned}$$

we put $\tau - a = (\sigma(x)-a)\sigma(s)$, this gives us that $\Delta\, x = \Delta(\sigma(x)) = \dfrac{1}{\sigma(s)}\Delta\,\tau$, by applied (3), we get

$$\begin{aligned}\int_a^\infty \left(\frac{G^\sigma(x)}{\sigma(x)-a}\right)^p \Delta\, x &\le \left(\int_0^1 \left[\frac{1}{\sigma(s)}\int_a^\infty g^{\,p}(\tau)\Delta\tau\right]^{\frac{1}{p}} \Delta\, s\right)^p \\ &= \left(\int_0^1 \sigma(s)^{-\frac{1}{p}} \Delta\, s\right)^p \int_a^\infty g^{\,p}(\tau)\Delta\tau \\ &\le \left(\frac{p}{p-1}\right)^p \int_a^\infty g^{\,p}(\tau)\Delta\tau.\end{aligned}$$

Remark. If we put $\mathbf{T} = \mathbb{R}$, we get the classical Hardy inequality.
If we take ($\mathbf{T} = \mathbf{Z}$), we deduce the discrete form of Hardy inequality.

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Chaos in the Fractional Lozi Map

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Abstract: In this paper, we propose a fractional map based on the integer-order unified map. The chaotic behaviour of the proposed map is analysed by means of bifurcations plots and some chaotic attractors that exist in the behaviour of our fractional map.
Keywords: Lozi map; Fractional-order map; Chaotic attractor.

1. Introduction

In 1976 the astronomer and mathematician Hénon proposed a two-dimensional iterated map as a simplified model of the Poincaré map for the Lorenz equations. This iterated map reads

$$H\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - ax^2 + by \\ x \end{pmatrix}. \tag{1}$$

The map (1) displays a chaotic attractor which appears to be the product of a one-dimensional manifold by a Cantor set. After Lozi introduced a two-dimensional map in 1978 where he replaced the quadratic term in the Hénon map (1) by a piecewise linear one [4]

$$L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - a \mid x \mid + by \\ x \end{pmatrix}. \tag{2}$$

This map displays a chaotic attractor for $a = 1.7$ and $b = 0.5$.
In the next section we give some fractional discrete-time calculus.

2. Fractional discrete-time calculus

Let $a \in \mathbb{R}$ fixed and let $\mathbb{N}_a = \{a, a+1, a+2, ...\}$ denotes the isolated time scale [2]. For the function $u(n)$, the delta difference operator Δ is defined as

$$\Delta u(n) = u(n+1) - u(n).$$

Definition 1. [3] Let $u : \mathbb{N}_a \longrightarrow \mathbb{R}$ and $v > 0$. Then, the fractional sum of v order is defined by

$$\Delta_a^{-v} u(t) = \frac{1}{\Gamma(v)} \sum_{s=a}^{t-v} (t - \sigma(s))^{(v-1)} u(s), \; t \in \mathbb{N}_{a+v}, \tag{3}$$

where a is the starting point, $\sigma(s) = s + 1$ and $t^{(v)}$ is the falling function defined in terms of the Gamma function as

$$t^{(v)} = \frac{\Gamma(t+1)}{\Gamma(t+1-v)}. \tag{4}$$

Definition 2. [4] For $v > 0$, $v \notin \mathbb{N}$ and $u(t)$ defined on $\mathbb{N}_a$, the Caputo-like delta difference is defined by

$$\begin{aligned} {}^c\Delta_a^v u(t) &= \Delta_a^{-(m-v)} \Delta^m u(t) \\ &= \frac{1}{\Gamma(m-v)} \sum_{s=a}^{t-(m-v)} (t - \sigma(s))^{(m-v-1)} \Delta_s^m u(s), \end{aligned}$$

where $t \in \mathbb{N}_{a+m-v}$, $m = [v] + 1$.

Theorem 1. [5] For the delta fractional difference equation

$$\begin{cases} {}^c\Delta_a^v u(t) = f(t + v - 1, u(t + v - 1)) \\ \Delta^k u(a) = u_k, m = [v] + 1, k = 0, \ldots, m - 1, \end{cases}$$

the equivalent discrete integral equation can be obtained as

$$u(t) = u_0(t) + \frac{1}{\Gamma(v)} \sum_{s=a+m-v}^{t-v} (t - \sigma(s))^{(v-1)} \times f(s + v - 1, \; u(s + v - 1), \; t \in \mathbb{N}_{a+m},$$

where

$$u_0(t) = \sum_{k=0}^{m-1} \frac{(t-a)^{(a)}}{k!} \Delta^k u(a).$$

3. The fractional-order version of the map (2).

The first order difference of (2) can be easily formulated as:

$$\begin{cases} \Delta x_n = 1 - a\,|x_n| - b y_n - x_n, \\ \Delta y_n = x_n - y_n. \end{cases} \tag{5}$$

Using the Caputo-like delta difference with d as the starting point, the fractional-order difference of (5) is given by

$$\begin{cases} {}^{c}\Delta_{a}^{v}x(t) = 1 - a\,|x(t-1+v)| - by(t-1+v) - x(t-1+v), \\ {}^{c}\Delta_{a}^{v}x(t) = x(t-1+v) - y(t-1+v), \end{cases} \tag{6}$$

for $0 < v \leq 1$ and $t \in \mathbb{N}_{d+1-v}$.
Following Theorem 1, using the discrete kernel function (7):

$$(t-\sigma(\mathrm{s}))^{v-1} = \frac{\Gamma(t-\mathrm{s})}{\Gamma(t-\mathrm{s}-v+1)}, \tag{7}$$

and assuming that $d = 0$, the numerical formulas for the fractional map (6) may be obtained as:

$$\begin{cases} x_n = x_0 + \frac{1}{\Gamma(v)} \sum\limits_{j=1}^{n} \frac{\Gamma(n-j+v)}{\Gamma(n-j+1)} (1 - a\,|x_{j-1}| - by_{j-1} - x_{j-1}), \\ y_n = y_0 + \frac{1}{\Gamma(v)} \sum\limits_{j=1}^{n} \frac{\Gamma(n-j+v)}{\Gamma(n-j+1)} (|x_{j-1}| - y_{j-1}). \end{cases} \tag{8}$$

For $v = 1$, the discrete fractional map (8) can be reduced to the classical one (2).
Assume $v = 0.8$, $X_0 = (0.3, 0.5)'$, then we can derive the numerical solutions X_n as in figure 1.

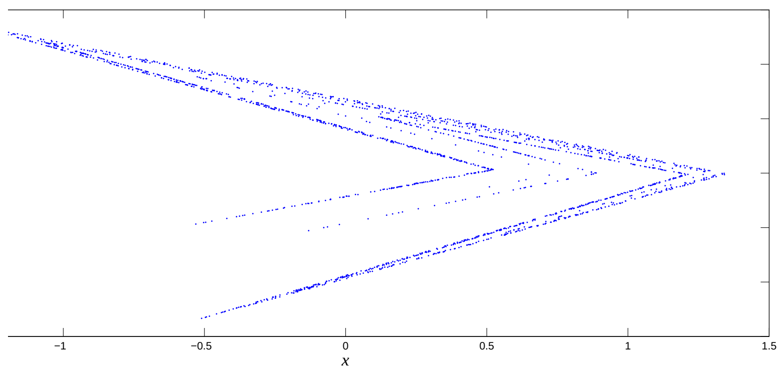

Fig.1-Chaotic attractor of the fractional map (8) obtained for $a = 1.4$, $b = -0.4$ and $v = 0.8$.

4. Conclusion

The chaotic behaviour of the fractional Lozi map is analysed by means of bifurcations diagram. Based on the obtained results, we see that the fractional order v has an impact on the existence and shape of the chaotic behaviour.

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Stability analysis and an optimal control applied to the spread of HIV/AIDS system

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Abstract: The aim of this study is to discuss the dynamics of HIV/AIDS model prosed by [5]. We have divided the total population into five classes, namely (susceptible individuals, infective individuals who do not know that they are infected, HIV positive individuals who know that they are infected and that of the AIDS population). We prove that the proposed model has two distinct equilibria (disease-free equilibrium and the positive endemic equilibrium).By using the Routh-Hurwitz criterion and the Descartes' rule of signs, we establish the local stability of the disease-free equilibrium subject to the basic reproduction number being smaller than to unity, on the other hand, the endemic equilibrium subject to the basic reproduction being greater than unity.

We have also discuss the previous model with three controls strategies of condom use u_1, screening of unawar infectives u_2 and treatment of unaware u_3.

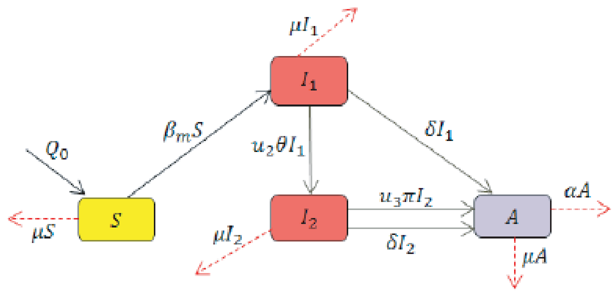


Figure 1: Flow Diagram for the HIV/AIDS disease transmission system with control.

Thus, the model is given by :

$$\frac{dS}{dt} = Q_0 - \beta_m S - \mu S$$

$$\frac{dI_1}{dt} = \beta_m S - (u_2\theta + \delta + \mu)I_1$$

$$\frac{dI_2}{dt} = u_2\theta I_1 - (\delta + \mu + u_3\pi)I_2$$

$$\frac{dA}{dt} = \delta I_1 + (\delta + u_3\pi)I_2 - (\alpha + \mu)A$$

where

$$\beta_m = \frac{(1-u_1)\left(\beta_1 c_1 I_1 + \beta_2 c_2 I_2 + \beta_3 c_3 A\right)}{N}.$$

The controls strategies aimed at controlling of the spread of HIV/AIDS epidemic.
The objective functional is defined as:

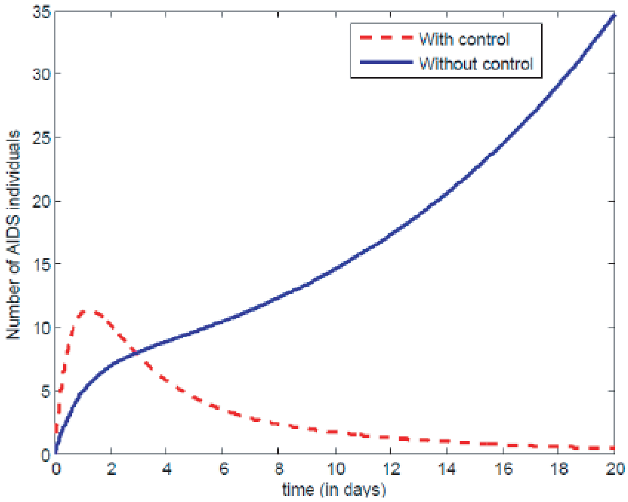
$$J(u_1, u_2, u_3) = \int_0^T \left(aI_1 + b_1 u_1^2 + b_2 u_2^2 + b_3 u_3^2\right) dt.$$

Our aim here is to minimize the number of unaware infectives I_1, while minimizing the cost control u_{12}, u_2 and u_3. Then we seek an optimal control u_1^*, u_2^* and u_3^* such that

$$(u_1^*, u_2^*, u_3^*) = \min\left\{J\left(u_1, u_2, u_3\right) : u_1, u_2 \text{ and } u_3 \in U\right\},$$

where U is the admissible control set defined by

$$U = \left\{(u_1, u_2, u_3) : 0 \leq u_i \leq 1,\ t \in [0, T],\ \text{for } i = 1, 2, 3\right\}.$$



Significance of the optimal control of the Aids individuals.

Figure 2: In this case, the initial values are (S(0),$I_1(0)$, $I_2(0)$, $A(0)$) = (800 ,40 , 45 ,0) with weight parameters a = 800, $b_1 = 35$, $b_2 = 55$, $b_3 = 75$. Other parameters are given by $Q_0 = 2000$, $\mu = 0.02$, $\theta = 0.02$, $\pi = 0.6$, $\delta = 0.1$, $\alpha = 1$, $\beta_1 = 0.20$, $\beta_2 = 0.15$, $\beta_3 = 0.12$.

The Pontryagin's maximum principle [2] and the existence result of optimal control [1] are used to characterize the optimal control.
Finally, the numerical simulation of both model i.e with control and without control, shows that this strategy helps to reduce the number of infected and the cost of control.
Keywords: Dynamical systems; Stability analysis; Optimal control; HIV/AIDS.

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Existence of solutions for a nonlinear fractional p-Laplacian boundary value problem

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Abstract: The main objective of this paper is to prove the existence of solutions for a fractional p-Laplacian boundary value problem $(P1)$ involving both the Riemann-Liouville and the Caputo types fractional derivatives:

$$(P1)\quad \begin{cases} -D_{1^-}^{\beta}\left(\phi_p\left(D_{0^+}^{\alpha}u\left(t\right)\right)\right)+f\left(t,u\left(t\right)\right)=0, & 1,0\leq t\leq 1 \\ u\left(0\right)=u'\left(0\right)=0, D_{0^+}^{\alpha}u(1)=0 \end{cases}$$

where $1<\alpha<2, 0<\beta<1$; $D_{0^+}^{\alpha}$, $D_{1^-}^{\beta}$are the standard Riemann-Liouville fractional derivatives, $\phi_p(s)=|s|^{p-2}s, p>1, \phi_p^{-1}=\phi_q, \frac{1}{p}+\frac{1}{q}=1$, u is the unknown function and $f\in C([0,1]\times\mathbb{R},\mathbb{R})$. Recently, the study of nonlinear fractional differential equations has attracted much attention of researchers and different methods have been investigated;
Besides, the p-Laplacian operator boundary-value problems have been studied in terms of their importance in theory and applications in mathematics, analyzing mechanics, physics and dynamic systems. However, there are many studies of the existence and uniqueness of boundary conditions of fractional differential equations with the p-Laplacian operator by many techniques.
By using the method of lower and upper solutions and the Schauder fixed point theorem, we prove the existence of solutions of problem $(P1)$.
To overcome the difficulties, we convert the problem $(P1)$ into an equivalent Caputo boundary value problem of order β, then we construct explicitly the upper and lower solutions of the problem $(P1)$, under some conditions on the nonlinear term f . We use Schauder fixed point theorem to prove the existence of solutions to the problem $(P1)$.
The method of upper and lower solutions has been applied in the investigation of the existence of solutions for nonlinear boundary value problems in many works.

Keywords: Fractional p-Laplacian, Boundary value problem, Method of upper and lower solutions, Existence of solutions.

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Invariant algebraic curves and the first integral of a class of Kolmogorov systems

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Abstract: In this work we introduce an explicit expression of invariant algebraic curves of the multi-parameter planar Kolmogorov systems (1).

$$\left\{ \begin{array}{l} x' = \dfrac{dx}{dt} = xF(x,y), \\ y' = \dfrac{dy}{dt} = yG(x,y), \end{array} \right. \tag{1}$$

where F , G are two functions in the variables x and y. then we proved that these systems are integrable and introduced an explicit expression for a first integral.

Keywords: Kolmogorov System, First Integral, Periodic Orbits, Limit Cycle

The autonomous differential system on the plane given by

$$\left\{ \begin{array}{l} x' = x \left(\begin{array}{c} 1 + ax^2 + bxy + cy^2 - (a+1)x^4 - bx^3y- \\ (c+n+2)x^2y^2 - mxy^3 - (s+1)y^4 \end{array} \right), \\ y' = y \left(\begin{array}{c} (1 + nx^2 + mxy + sy^2 - (a+1)x^4 - bx^3y- \\ (c+n+2)x^2y^2 - mxy^3 - (s+1)y^4), \end{array} \right) \end{array} \right\} \tag{2}$$

where a, b, c, n, m and s are real constants, the derivatives are performed with respect to the time variable. Is frequently used to model the iteration of two species occupying the same ecological niche. There are many natural phenomena which can be modeled by the Kolmogorov systems such as mathematical ecology and population dynamics chemical reactions, plasma physics, hydrodynamics, economics, etc. In the qualitative theory of planar dynamical systems, one of the most important topics is related to the second part of the unsolved Hilbert 16th problem.

Main result

Our main result is contained in the following theorem.

Theorem . Consider a multi-parameter planar Kolmogorov system (2), then the following statements hold.

(h_1) The curve $U(x,y) = xy(nx^2+mxy+sy^2)-xy(ax^2+bxy+cy^2)$ is an invariant algebraic curve of system (2).

(h_2) If $f_3(\theta) \neq 0$, then the system (2) has the first integral

$$H(x,y) = \frac{\exp\left(\int_0^{\arctan\frac{y}{x}} D(w)dw\right) + (x^2+y^2-1)\int_0^{\arctan\frac{y}{x}} \exp(-\int_0^s D(w)dw)C(s)ds}{x^2+y^2-1}.$$

Moreover the phase portrait of the differential system (2), in Cartesian coordinates is given by

$$x^2+y^2 = \frac{h+\exp\left(\int_0^{\arctan\frac{y}{x}} D(w)dw\right) - \int_0^{\arctan\frac{y}{x}} \exp(-\int_0^s D(w)dw)C(s)ds}{h-\int_0^{\arctan\frac{y}{x}} \exp(-\int_0^s D(w)dw)C(s)ds}.$$

where $h \in \mathbb{R}$.

(h_3) If $f_3(\theta) = 0$ for all $\theta \in \mathbb{R}$, then the system (2) has the first integral $H(x,y) = \frac{y}{x}$. Moreover the phase portrait of the differential system (2), in Cartesian coordinates is given by $y - hx = 0$, where $h \in \mathbb{R}$.

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On the behavior of the solutions of a system of difference equations of second order defined by homogeneous functions

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Abstract: The aim of this work [2] is to study a general system of difference equations defined by continuous and homogeneous functions of degree zero. We establish results on local stability of the unique equilibrium point and to deal with the global attractivity, and so the global stability, some general convergence theorems are provided. Necessary and sufficient conditions on existence of prime period two solutions of our system are given. Also, a result on oscillatory solutions is proved. As applications of the obtained results, concrete models are investigated. Our system generalize some existing works in the literature, see for example [1] and our results can be applied to study new models of systems of difference equations.

Keywords: Homogeneous functions, systems of difference equations, local and global stability, periodicity, oscillatory solutions.

Introduction

In the last three decades, a lot of studies are devoted to the subject of difference equations. A huge number of models of difference equations investigated by researchers are defined by particular homogeneous functions. In the present talk, we will present our obtained results [2] on the following general system of difference equations defined by

$$x_{n+1} = f(y_n, y_{n-1}),\ y_{n+1} = g(x_n, x_{n-1}) \tag{1}$$

where $n \in \mathbb{N}_0$, the initial values x_{-1}, x_0, y_{-1} and y_0 are positive real numbers, the functions $f,\ g :\ (0, +\infty)^2 \to (0, +\infty)$ are continuous and homogeneous of degree zero.
Clearly if we take $y_{-i} = x_{-i}$, $i = 1, 2$, and $g \equiv f$, then the System (1), will be

$$x_{n+1} = f(x_n, x_{n-1}). \tag{2}$$

The behavior of the solutions of Equation (2), can be founded in [1]. In particular, our results generalize and complete those in [1]. Also, for particular choices of the functions f

and g, we can recover new difference equations and systems.

1. On the stability of the unique equilibrium point

In this part, we give conditions for the stability of the unique equilibrium point of of System (1).

Theorem 1.1. Assume that $f(u,v)$, $g(u,v)$ are C^1 on $(0,+\infty)^2$. The equilibrium point

$$(\overline{x},\overline{y}) = (f(1,1), g(1,1))$$

of System (1) is locally asymptotically stable if

$$\left|\frac{\partial f}{\partial u}(1,1).\frac{\partial g}{\partial u}(1,1)\right| < \frac{f(1,1).g(1,1)}{4}.$$

In order to prove that the equilibrium point is a global attractor, we will prove the following general convergence theorems.

Theorem 1.2. Consider System (1). Assume that the following statements are true:

1. H_1: There exist a, b, α, $\beta \in (0,+\infty)$ such that

$$a \leq f(u,v) \leq b,\ \alpha \leq g(u,v) \leq \beta,\ \forall (u,v) \in (0,+\infty)^2.$$

2. H_2: $f(u,v)$, $g(u,v)$ are increasing in u for all v and decreasing in v for all u.

3. H_3: If $(m_1, M_1, m_2, M_2) \in [a,b]^2 \times [\alpha,\beta]^2$ is a solution of the system

$$m_1 = f(m_2, M_2),\ M_1 = f(M_2, m_2),\ m_2 = g(m_1, M_1),\ M_2 = g(M_1, m_1)$$

then

$$m_1 = M_1,\ m_2 = M_2.$$

Then every solution of System (1) converges to the unique equilibrium point

$$(\overline{x},\overline{y}) = (f(1,1), g(1,1)).$$

Theorem 1.3. Consider System (1). Assume that the following statements are true:

1. H_1: There exist a, b, α, $\beta \in (0,+\infty)$ such that
$$a \leq f(u,v) \leq b,\ \alpha \leq g(u,v) \leq \beta,\ \forall (u,v) \in (0,+\infty)^2 .$$
2. H_2: $f(u,v)$, $g(u,v)$ are decreasing in u for all v and increasing in v for all u.
3. H_3: If $(m_1, M_1, m_2, M_2) \in [a,b]^2 \times [\alpha,\beta]^2$ is a solution of the system
$$m_1 = f(M_2, m_2),\ M_1 = f(m_2, M_2),\ m_2 = g(M_1, m_1),\ M_2 = g(m_1, M_1)$$
then
$$m_1 = M_1,\ m_2 = M_2.$$
Then every solution of System (1) converges to the unique equilibrium point
$$(\overline{x}, \overline{y}) = (f(1,1), g(1,1)).$$

Theorem 1.4. Consider System (1). Assume that the following statements are true:

1. H_1: There exist a, b, α, $\beta \in (0,+\infty)$ such that
$$a \leq f(u,v) \leq b,\ \alpha \leq g(u,v) \leq \beta,\ \forall (u,v) \in (0,+\infty)^2 .$$
2. H_2: $f(u,v)$ is increasing in u for all v and decreasing in v for all u, however $g(u,v)$ is decreasing in u for all v and increasing in v for all u.
3. H_3: If $(m_1, M_1, m_2, M_2) \in [a,b]^2 \times [\alpha,\beta]^2$ is a solution of the system
$$m_1 = f(m_2, M_2),\ M_1 = f(M_2, m_2),\ m_2 = g(M_1, m_1),\ M_2 = g(m_1, M_1)$$
then
$$m_1 = M_1,\ m_2 = M_2.$$
Then every solution of System (1) converges to the unique equilibrium point
$$(\overline{x}, \overline{y}) = (f(1,1), g(1,1)).$$

Theorem 1.5. Consider System (1). Assume that the following statements are true:

1. H_1: There exist a, b, α, $\beta \in (0, +\infty)$ such that
$$a \leq f(u, v) \leq b,\ \alpha \leq g(u, v) \leq \beta,\ \forall (u, v) \in (0, +\infty)^2 .$$
2. H_2: $f(u, v)$ is decreasing in u for all v and increasing in v for all u, however $g(u, v)$ is increasing in u for all v and decreasing in v for all u.
3. H_3: If $(m_1, M_1, m_2, M_2) \in [a, b]^2 \times [\alpha, \beta]^2$ is a solution of the system
$$m_1 = f(M_2, m_2),\ M_1 = f(m_2, M_2),\ m_2 = g(m_1, M_1),\ M_2 = g(M_1, m_1)$$
then
$$m_1 = M_1,\ m_2 = M_2.$$
Then every solution of System (1) converges to the unique equilibrium point
$$(\overline{x}, \overline{y}) = (f(1, 1), g(1, 1)).$$

Now, we are able to state our result on the global stability of the unique equilibrium point $(\overline{x}, \overline{y}) = (f(1, 1), g(1, 1))$ of System (1).

Theorem 1.6. Under the assumptions of Theorem 1.1 and the assumptions of Theorem 1.2 or Theorem 1.3 or Theorem 1.4 or Theorem 1.5, the equilibrium point $(\overline{x}, \overline{y}) = (f(1, 1),\ g(1, 1))$ is globally stable.

2. Existence of periodic and oscillatory solutions

Here we present some results on existence of periodic and oscillatory solutions.

Theorem 2.1. Assume that $(\alpha - 1)(\beta - 1) \neq 0$. Then, System (1) have a prime period two solution
$$..., (\alpha p, \beta q),\ (p, q), (\alpha p, \beta q),\ (p, q), ...$$
if and only if
$$f(1, \beta) = \alpha f(\beta, 1),\ g(1, \alpha) = \beta g(\alpha, 1),$$
where
$$p = f(\beta, 1),\ q = g(\alpha, 1).$$

Theorem 2.2. Let $(x_n, y_n)_{n=-1,0,...}$ be a solution of System (1) and assume that $f(x, y)$, $g(x, y)$ are decreasing in x for all y and are increasing in y for all x.

1. If
$$x_0 < \overline{x},\ x_{-1} > \overline{x},\ y_0 < \overline{y},\ y_{-1} > \overline{y},$$
then we get
$$x_{2n} < \overline{x},\ x_{2n-1} > \overline{x},\ y_{2n} < \overline{y},\ y_{2n-1} > \overline{y},\ n \in \mathbb{N}_0.$$
That is for both $(x_n)_{n\geq -1}$ and $(y_n)_{n\geq -1}$ we have semi-cycles of length one of the form
$$+-+-+-\cdots.$$

2. If
$$x_0 > \overline{x},\ x_{-1} < \overline{x},\ y_0 > \overline{y},\ y_{-1} < \overline{y},$$
then we get
$$x_{2n} > \overline{x},\ x_{2n-1} < \overline{x},\ y_{2n} > \overline{y},\ y_{2n-1} < \overline{y},\ n \in \mathbb{N}_0.$$
That is for both $(x_n)_{n\geq -1}$ and $(y_n)_{n\geq -1}$ we have semi-cycles of length one of the form
$$-+-+-+\cdots.$$

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Study of the dynamics of a biomathematic model governed by a system of ordinary differential equations

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Abstract:
The present work discusses a prey-predator model where the prey species are subjected to a harvest. The model is a modified version from the classic Lotka-Volterra predator-prey model. The equilibria of the model are obtained and the dynamical behaviors of the proposed system are examined. Simulations of the model are performed.
Keywords: Predator-prey model; Harvesting; positive equilibria; limit cycle.

Introduction
Many researchers are interested to the dynamic of predator-prey interactions models and explored the processes that affect them. The interaction between a predator and prey may be modeled by the classical Lotka-Voltera model

$$\begin{cases} \dot{x} = rx(1 - \dfrac{x}{k}) - axy \\ \dot{y} = y(-d + cx) \end{cases} \tag{1}$$

Where x and y represent the prey and predator species, respectively; r, k, a, c, and d are positive constants. In the absence of the predation, the prey grows logistically with intrinsic growth rate r and carrying capacity k. In the presence of the predation, the prey species decreases at a rate proportional to the functional response ax, where a presents the rate of predation. The factor c denotes the rate of growth of the predator due to its predation. Without the prey, no predation occurs and the predator species decreases exponentially with mortality rate d.
To enrich the model (1), several researchers modified the nonlinear functional response and added some other elements such as, Pollution, toxicity, refuge,...etc.
As harvesting is an important and effective method to prevent and control the explosive growth of predators or prey when they are enough, it is reasonable and necessary

to introduce the harvest of populations into models. We then focus in this work on the predator-prey model with harvest.

1 Model formulation

Using model (1) as our baseline model, we assume that harvesting takes place, but only the prey population is under harvesting and introduce harvesting function $H(x)$ of the prey to prey-predator model (1) for discussing its dynamical features. The interactive dynamics are governed by the following system

$$\begin{cases} \dot{x} = rx(1-\frac{x}{k}) - axy - H(x) \\ \dot{y} = y(-d+cx) \end{cases} \tag{2}$$

Where

$$H(x) = \begin{cases} mx & if\ 0 \leq x \leq x_0 \\ h & if\ x > x_0 \end{cases}$$

We assume that the harvesting rate is proportional to the predator population size until it reaches a threshold value. The harvesting rate will then be kept as a constant. Denote the harvesting threshold value as $h = mx_0$.

2 Preliminary results

We can show that solutions of system (2) with positive initial conditions are all positive for $t > 0$ and uniformly bounded. Thus the following set:

$$S = \left\{ (x,y) \in \mathbb{R}^2_+, \quad cx + ay \leq \frac{ck}{4rd}(r+d)^2 \right\}$$

is positive invariant for system (2).

We are only interested in analyzing the solutions of system (2) in the first quadrant $\mathbb{R}^2_+$. The equilibria of this system in the subregion of S with $0 \leq x \leq x_0$ are
$P_0 = (0,0)$, $P_1 = (k(1-\frac{m}{r}), 0)$, $P^* = \left(\frac{d}{c}, \frac{r}{a}(1-\frac{d}{ck}) - \frac{m}{a}\right)$

Theorem 1.
1) Equilibrium P_0 is instable if $m > r$, and stable if $m < r$.

2) Equilibrium P_1 is instable if $m < r(1-\frac{d}{ck})$, and stable if $m > r(1-\frac{d}{ck})$; when $m = r(1-\frac{d}{ck})$ there may exist a bifurcation at P_1.
3) The positive equilibrium P^* of system (2) is globally asymptotically stable if $m < r(1-\frac{d}{ck})$ *and* $x_0 \geq \frac{d}{c}$.

We denote $h_1 = \frac{rk}{4}$, $h_2 = \frac{rd}{c}\left(1-\frac{d}{ck}\right)$, $h_3 = rx_0\left(1-\frac{x_0}{k}\right)$, $\hat{h} = \frac{rd^2}{kc^2}$.

Proposition 1.
In the subregion of S with $x > x_0$, system (2) has,
1) No positive equilibrium, if $h > h_1$.
2) A unique positive equilibrium $q_0 = (x_0, 0)$ with $x_0 = \dfrac{k}{2}$, if $h = h_1$
3) Two positive equilibria $q_1 = (x_1, 0)$ and $q_2 = (x_2, 0)$ with
$x_1 = \frac{rk-\sqrt{rk(rk-4h)}}{2r}$ and $x_2 = \frac{rk+\sqrt{rk(rk-4h)}}{2r}$,
if $h_2 < h < h_1$ and $h_3 < h$
4) Three positive equilibria $q_1 = (x_1, 0)$, $q_2 = (x_2, 0)$ and $q^* = (x^*, y^*)$ with
$x_1 = \frac{rk-\sqrt{rk(rk-4h)}}{2r}$, $x_2 = \frac{rk+\sqrt{rk(rk-4h)}}{2r}$,
$x^* = \frac{d}{c}$ and $y^* = \frac{r}{a}\left(1-\frac{d}{ck}\right) - \frac{ch}{ad}$,

if $h < h_1$, $x_0 < \frac{d}{c}$ and $h_3 < h < h_2$.

Theorem 2.
1) q_1 is an unstable node if $h > h_2$ and q_1 is a saddle point if $h < h_2$.
2) If $h < h_2$, then q_2 is a saddle point, when $h > h_2$ is stable node.
3) If $\hat{h} < h < h_2$, q^* is an unstable focus or node, and when $h < min\left\{h_2, \hat{h}\right\}$, q^* is a stable focus or node.

We have the main result of this work:
Theorem 3.
Suppose $\hat{h} < h < h_2$, then system (2) has at least a limit cycle which encircles q^*.

Conclusion

In this work, we have performed a mathematical analysis to our model, we thus examined the dynamics of the proposed model, we studied the existence and the global stability of the equilibrium states. We have also shown that model (**??**) has at least one limit cycle. Finally, a numerical simulation is performed to verify the theoretical results obtained.

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Ordinary differential equations and continuous dynamical systems

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Etude d'un problème à conditions aux limites non locales généralisées de type Bitsadze-Samarskii dans les espaces L^p

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Résumé : Ce travail est consacré à l'étude d'un problème elliptique de type Bitsadze-Samarskii dans le cadre des espaces de Banach UMD. Ici, on trouve des résultats concernant l'existence, l'unicité et la régularité de la solution. On définit deux types de solutions (solution stricte et semi-stricte) et on donne des conditions nécessaires et suffisantes sur les données pour obtenir ces résultats.
Mots clés: Conditions aux limites non locales, semi-groupe analytique, puissance imaginaire d'opérateur, espace UMD.

Dans ce travail, on étudie un problème elliptique aux limites non locales de type Bitsadze-Samarskii dans le cadre L^p.
Soit $x_0 \in [0,1[$, on considère le problème suivant:

$$(P1) : \begin{cases} -u''(x) + Au(x) = f(x), \text{ p.p. } x \in]0,1[\\ u(0) = u_0, \\ u(1) - Hu(x_0) = u_{1,x_0}, \end{cases}$$

o $f \in L^p(0,1;X)$, $1 < p < +\infty$, X est un espace de Banach complexe UMD, u_0 et u_{1,x_0} sont des éléments de X, A est un opérateur linéaire fermé de domaine $\mathcal{D}(A)$ dans X et H est un opérateur linéaire fermé de domaine $\mathcal{D}(H)$ dans X.

Allaberen Ashyralyev [1] s'est intéressé (en 2008) au problème (P1), pour $H = \alpha I$, $(\alpha > 0)$, dans le cadre des espaces de Hlder et a montré que ce problème est bien posé, en vérifiant l'inégalité de coercivité.

On cherche (sous quelques hypothèses) deux types de solutions:

- Solution semi-stricte, c'est à dire u vérifie (P1) et:

$$u \in W^{2,p}(0,1-\varepsilon;X) \cap L^p(0,1-\varepsilon;\mathcal{D}(A)) \text{ et } u' \in L^p(0,1;X).$$

- Solution stricte, c'est à dire u vérifie (P1) et:

$$u \in W^{2,p}(0,1;X) \cap L^p(0,1;\mathcal{D}(A)).$$

La méthode est basée essentiellement sur la constuction d'une représentation de la solution, l'utilisation des semi-groupes, les domaines fractionnaires des opérateurs, les espaces d'interpolation et la théorie des sommes d'opérateurs en s'inspirant du travail de H. Hammou et al [2].

On trouve finalement, des résultats concernant l'existence, l'unicité et la régularité de la solution de ce problème. Voir B. Hamdi et al [3].

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Existence of solution for a third-order boundary value problem

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Abstract: We study the existence of solutions for the third-order boundary value problem (BVP) having the following form

$$-u^{'''}(t) + f(t, u(t)) = 0, \quad 0 < t < 1,$$
$$u(0) = u^{'}(0) = u^{''}(1) = 0.$$

The boundary value problem is very similar type considered. It is assumed that f is a function from the space $C([0, 1] \times \mathbb{R}, \mathbb{R})$. The main tool used in the proof is the Leray-Schauder nonlinear alternative. As an application, we also given an example to illustrate the results obtained.

Keywords: Green's function, Nontrivial solution, Leary-Schauder nonlinear alternative, Fixed point theorem, Boundary value problem.

1. Preliminaries

We consider the BVP under the assumption that $f \in C([0, 1] \times \mathbb{R}, \mathbb{R})$. $E = C^3[0, 1]$ with the norm $\|u\| = \max |u|_\infty$ where $|u|_\infty = \max_{t\in[0,1]} |u(t)|$ for any $u \in E$.

Lemma 1. Let $y \in C([0, 1])$. Then the three-point BVP

$$-u^{'''}(t) + y(t) = 0, \quad 0 < t < 1,$$
$$u(0) = u^{'}(0) = u^{''}(1) = 0,$$

is equivalent to the integral equation

$$u(t) = \int_0^1 G(t, s)y(s)ds$$

where $G : [0,1] \times [0,1] \to [0,\infty)$ denotes the Green function given by

$$G(t,s) = \frac{1}{2}\begin{cases} t^2, & 0 \le t \le s \le 1, \\ s(2t-s), & 0 \le s \le t \le 1, \end{cases}$$

Define the integral operator $T : E \to E$ by

$$Tu(t) = \frac{1}{2}\int_0^t s(2t-s)y(s)ds + \frac{1}{2}\int_t^1 t^2 y(s)ds.$$

Lemma 2. Let E be a Banach space and Ω be a bounded open subset of E, $0 \in \Omega$. $T : \overline{\Omega} \to E$ be a completely continuous operator. Then, either

(i) there exists $u \in \partial\Omega$ and $\lambda > 1$ such that $T(u) = \lambda u$, or

(ii) there exists a fixed point $u^* \in \overline{\Omega}$ of T.

2. Existence of nontrivial solutions

In this section, we present two theorems for prove the existence of a nontrivial solution for the BVP. Suppose that $f \in C([0,1] \times \mathbb{R}, \mathbb{R})$.

Theorem 1. Suppose that $f(t,0) \neq 0$ and there exist nonnegative functions $k, h \in L^1[0,1]$ such that

$$|f(t,u)| \le k(t)|u| + h(t), \quad a.e. \ (t,u) \in [0,1] \times \mathbb{R},$$

and

$$\int_0^1 sk(s)ds < 1.$$

Then the BVP has at least one nontrivial solution $u^* \in E$.

Theorem 2. Suppose that $f(t,0) \neq 0$, and there exist nonnegative functions $k, h \in L^1[0,1]$ such that

$$|f(t,u)| \le k(t)|u| + h(t), \quad a.e. \ (t,u) \in [0,1] \times \mathbb{R}.$$

Assume that one of the following conditions holds

(1) There exists a constant $\alpha > -2$ such that

$$k(s) \le (2+\alpha)s^{\alpha}, \quad a.e. \quad s \in [0,1],$$

$$meas\{s \in [0,1] : k(s) < (2+\alpha)s^{\alpha}\} > 0.$$

(2) There exists a constant $\alpha > -1$ such that

$$k(s) \leq (1+\alpha)(2+\alpha)(1-s)^{\alpha}, \quad a.e. \quad s \in [0,1],$$

$$meas\{s \in [0,1] : k(s) < (1+\alpha)(2+\alpha)(1-s)^{\alpha}\} > 0.$$

(3) There exist a constant $p > 1$ such that

$$\int_0^1 k(s)^p ds < [(1+q)^{1/q}]^p, \qquad (\frac{1}{p} + \frac{1}{q} = 1).$$

Then the BVP has at least one nontrivial solution $u^* \in E$.

Example . Consider the following problem

$$\begin{cases} -u^{'''} + \frac{t}{7}|u| \cos\sqrt{u} + \frac{t^2}{4}u^2 + te^t + 1 = 0, \quad 0 < t < 1, \\ \\ u(0) = u^{'}(0) = u^{''}(1) = 0. \end{cases}$$

Set

$$f(t,u) = \frac{t}{7}|u| \cos\sqrt{u} + \frac{t^2}{4}u^2 + te^t + 1,$$

$$k(t) = \frac{t}{3} + t^2, \quad h(t) = te^t + 1.$$

It is easy to prove that $k, h \in L^1[0,1]$ are nonnegative functions, and

$$|f(t,u)| \leq k(t)|u| + h(t), \quad a.e. \ (t,u) \in [0,1] \times \mathbb{R}.$$

Moreover, we have

$$M = \int_0^1 sk(s)ds = \int_0^1 s(\frac{s}{3} + s^2)ds = \frac{1}{9} + \frac{1}{4} < 1.$$

Hence, by Theorem 1, the BVP (1) has at least one nontrivial.
solution u^* in E.

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On Monotone Generalized $\lambda - \alpha$-Nonexpansive Mappings in Banach Spaces with Applications to $L_1([0, 1])$

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Abstract: In the work i wish to expose, we introduce a new class of generalized monotone $(\lambda-\alpha)$ -nonexpansive mappings in which fixed point existence results and weak and strong convergence results are established.

We establish these results in the Banach spaces for the iteratif processes of Krasnoselskii and Mann. We have also considered an application to the space $L_1([0, 1])$ which has the particularity to be convex and not to be uniformly convex. In fact, in this work, we first established the theorems of the existence of fixed points in uniformly convex Banach spaces in all directions and, on the other hand, a strong and weak convergence for the iterative sequence of Krasnoselskii. Finally, we have finished this work with an application to Lebesgue's space $L_1([0, 1])$. Thus our results generalize and unify the relative results in the literature.

Key-words: Fixed point, Krasnoselskii iteration process, generalized monotone $(\lambda - \alpha)$ -nonexpansive mappings.

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Solutions multiples pour un problème aux limites posé sur la demi-droite réelle par la théorie de Morse

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Résumé : Ce papier est consacré à l'étude de l'existence d'une multiplicité de solutions pour un problème aux limites impulsif en utilisant la théorie de Morse et les groupes critiques à l'infinie.

Mots clés: Problèmes aux limites impulsif, demi-droite réelle, points critiques, groupes critiques, théorie de Morse.

Introduction

Dans ce travail nous nous intéressons à l'étude du problème suivant

$$\left\{\begin{array}{rcl} -(p(t)u'(t))' &=& q(t)f(t,u(t)), \quad t \neq t_j,\ j\in\{1,2,\ldots\},\ t>0, \\ u(0)=u(+\infty) &=& 0, \\ \Delta(p(t_j)u'(t_j)) &=& h(t_j)I_j(u(t_j)), \quad j\in\{1,2,\ldots\}, \end{array}\right. \tag{1}$$

o $f \in C([0,+\infty[\times \mathbb{R}, \mathbb{R})$, et satisfait:

$$|f(t,u)| \le c(1+|u|^{\alpha-1}), \forall t \ge 0, u \in \mathbb{R} \tag{2}$$

pour $c>0$ et $\alpha \in (2,+\infty)$, avec $f(t,0)=0, \forall t \in \mathbb{R}^+$ $q \in L^1((0,+\infty),\mathbb{R}^+)$, $q>0$, p.p., et tels que

$$M_1 = \int_0^{+\infty}\left(\int_t^{+\infty}\frac{ds}{p(s)}\right)dt < \infty \text{ et } M_2 = \int_0^{+\infty} q(t)\left(\int_t^{+\infty}\frac{ds}{p(s)}\right)dt < \infty.$$

$I_j \in C(\mathbb{R}, \mathbb{R}),\ j = \{1, 2, \ldots\}$ sont les fonctions impulsives et $t_0 = 0 < t_1 < t_2 < \ldots < t_j < \ldots < t_m \to +\infty$, lorsque $m \to +\infty$, sont les points d'impulsions qui sont en nombre infinie.

Nous supposerons que la fonction I_j satisfait la condition suivante.

$$\exists k > 0, \sigma \in (2, +\infty); |I_j(u)| \leq k|u|^{\sigma}, \forall u \in \mathbb{R}. \tag{3}$$

Posons

$$\triangle(p(t_j)u'(t_j)) = p(t_j^+)u'(t_j^+) - p(t_j^-)u'(t_j^-),$$

o $u'(t_j^+) = \lim\limits_{t \overset{>}{\to} t_j} u'(t)$ et $u'(t_j^-) = \lim\limits_{t \overset{<}{\to} t_j} u'(t)$ représente les limites à droite et à gauche de u' at t_j, respectivement. Finalement $h : \mathbb{R}^+ \to \mathbb{R}^+$ est une fonction tel que $\sum_{j=1}^{\infty} |h(t_j)| < \infty$.

Premier résultat

Supposons que les conditions suivantes sont satisfaites (H_1) Il existe $\delta > 0$, $\mu \in (0, 2)$ et $c_0 > 0$ tel que

$$F(t, u) \geq c_0|u|^{\mu},\ \text{pour } t \geq 0, |u| \leq \delta.$$

(H_2) (i) $\lim_{|u| \to \infty}(F(t, u) - \frac{1}{2}\lambda_1|u|^2) = -\infty$ uniformément pour $t \in [0, +\infty)$
(ii) $\exists \widehat{C} \geq 0, F(t, u) - \frac{\lambda_1}{2}|u|^2 \leq \widehat{C}, \forall t \in [0, +\infty), \forall u \in \mathbb{R}$ (I_1) $\exists C' \in \mathbb{R}, \int_0^u I_j(s)ds \geq C', \forall u \in \mathbb{R}$. Alors le problème (1) admet au moins une solution non triviale.

Deuxième résultat

En plus des conditions (H_2) et (I_1), supposons que les conditions suivantes sont satisfaites:
(H_3) Ils existent $\delta > 0, \lambda \in (\lambda_1, \overline{\lambda})$ tel que

$$\lambda_1|u|^2 \leq 2F(t, u) \leq \lambda|u|^2 \text{ pour tout } t \in [0, +\infty), |u| \leq \delta$$

(I_2) $\exists \delta > 0, \int_0^u I_j(s)ds = 0, \forall u \leq \delta$.
Then the problem (1) has at least two nontrivial solution.

Conclusion

La théorie de Morse nous a permit d'obtenir deux ou trois solutions en tenant compte de la solution triviale.

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GLOBAL PHASE PORTRAITS OF QUADRATIC SYSTEMS HAVING REDUCIBLE INVARIANT CURVE

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Abstract: In this paper, we calssify the global phase portraits of all quadratic polynomial differential systems having the invariant reducible cubic curve $(y-k)(x^2+y^2-1)=0$. We prove that there are 12 different topological phase portraits for such quadratic polynomial differential systems.

Keywords: Phase portrait, Poincaré disc, Reducible cubic.

Introduction

Let $\mathbb{R}[x;y]$ be the ring of real polynomials in the variables x and y. In this work we are interesting in studying the quadratic polynomial differential systems which are written as

$$\dot{x}=P(x,y)=P_0+P_1+P_2, \qquad \dot{y}=Q(x,y)=Q_0+Q_1+Q_2. \tag{1}$$

where P_i and Q_i are real polynomials of degree i where $i=0,\ 1,\ 2,$ in the variables $(x;y)$ and $P_2^2+Q_2^2\neq 0$.

This kind of differential systems is the simplest nonlinear polynomial systems, which appear in several branches of science, mainly in chemistry, physics, in population dynamics, hydrodynamics, biology, etc.

From 1960's, many researchers have been interesting in classifying the global phase portraits of quadratic systems which are not easy to be studied. In this work; we study the phase portraits of the quadratic polynomial differential systems

$$\begin{aligned}\dot{x}&=\frac{1}{2}(a-1)x^2+\frac{1}{2}(a-3)y^2+ky+\frac{1-a}{2},\\ \dot{y}&=xy-kx.\end{aligned} \tag{2}$$

These differential systems having the reducible cubic invariant algebraic curve of degree 3

$$H(x,y)=(y-k)(x^2+y^2-1)=0$$

Our main result is the following

Theorem. The polynomial differential systems (2) with $k \neq 0$ have 12 global phase portraists in the Poincaré disc topologically non equivalent.

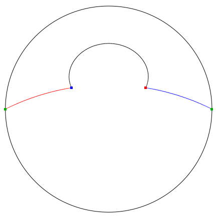

Figure 3:

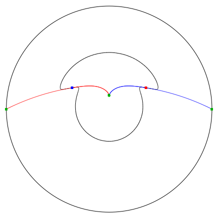

Figure 4:

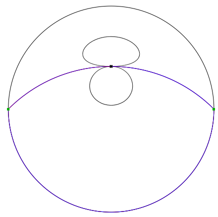

Figure 5:

Finite and infinite singularities

For the quadratic systems 2 their singular points are characterized in the following result.

Proposition . The following statements hold for the quadratic systems 2.

1 If $a \in (2-\sqrt{1-k^2}, 2+\sqrt{1-k^2})$ and $k \in (0,1)$, systems (2) have two hyperbolic finite singular points, a stable node at $(-\sqrt{1-k^2}, 0)$ and an unstable node at $(\sqrt{1-k^2}, 0)$. In the local chart U_1 it have one infinite singularity at $(0,0)$ which is a saddle, and the origin of the local chart U_2 is not a singularity of these systems.

2 If $a = 2 - \sqrt{1-k^2}$ and $k \in (0,1)$, systems (2) have three finite singular points: a stable node at $(-\sqrt{1-k^2}, 0)$, an unstable node at $(\sqrt{1-k^2}, 0)$ and a nilpotent singularity at $\left(0, \frac{1-\sqrt{1-k^2}}{k}\right)$, and its local phase portrait formed by two hyperbolic sectors. In the local chart U_1 this systems have one infinite singularity at $(0,0)$ which is a saddle and the origin of the local chart U_2 is not a singularity of these systems.

3 If $a = 2$ and $k = 1$, system (2) has one finite singularity at $(0,1)$ which is a linearly zero, and its local phase portrait formed by two elliptic sectors. In the local chart U_1

it has one infinite singularity at $(0,0)$ which is a saddle and the origin of the local chart U_2 is not a singularity of this system.

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Limit cycles of a family of discontinuous piecewise linear differential systems separated by conics

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Abstract: This paper is devoted to provide the maximum number of crossing limit cycles of two families of discontinuous piecewise linear differential systems. More precisely, we prove that the systems formed by two zones, such that, in one zone we define a linear Hamiltonian differential system without equilibrium point and in the second zone we define a linear differential center, can exhibit at most three crossing limit cycles having two intersection points with the conics of separation.
After we prove that the systems formed by three zones, where, in two no–adjacent zones we define the same class of differential system, and in the third zone we define another class, can exhibit three crossing limit cycles having four or two or four simultaneously intersection points with the conics of separation.
Keywords: limit cycles, discontinuous piecewise linear differential systems,linear centers, linear Hamiltonian systems, conics.

One of the main and difficult problems in the qualitative theory of a piecewise linear differential systems is the existence and the determination of limit cycles.
We recall that a limit cycle of a differential system is an isolated periodic orbit in the set of all periodic orbit of this system. Piecewise linear differential systems revent to Andronov, Vitt and Khaikin. Owing to the simplicity of this kind of differential systems, researchers had given a big interest on studying them and they have extensively a large relevance in the domain of engenering sciences, for example, we can modeled key of component in even simple electronic circuit, also the Diodes and transistors as a piecewise linear differential systems.
Even now, many papers devoted to study the existence and the number of limit cycles of these systems when the curve of separation is either a straight line, or an algebraic curves,

such that a conic or reducible or irreducible cubic curves, see [1,2].
Especially, in [3] Benterki and Llibre studied the existence of limit cycles of planar piecewise linear Hamiltonian systems without equilibrium points, where they proved that if these systems are separated by a parabola, a hyperbola or an ellipse; they can have at most 2, 3 or 3 crossing limit cycles, respectively.
In [4] Damene and Benterki provided the maximum number of crossing limit cycles of two different families of discontinuous piecewise linear differential systems separated by a cubic curves.
In our paper we are going to consider two families of planar linear differential systems, the first one is Hamiltonians without equilibrium points and the second one is a family of centers, and the curves of separation curve are conics.

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Limit Cycles For A Class of Generalized Kukles Differential Systems

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Résumé : Dans ce travail, en utilisant la théorie de moyennisation du premier et du deuxime ordre, nous étudions le nombre maximal de cycles limites qui bifurquent des orbites périodiques du centre linéaire d'une classe de systémes différentiels de Kukles généralisée.
Mots clés: théorie de moyennisation, Systme différentielle de Kukles, cycle limite.

Introduction

L'un des principales sujets dans la théorie des équations différentielles est l'étude des cycles limites : leur existence, leur nombre et leur stabilité. Un cycle limite d'une équation différentielle est une orbite périodique isolée dans l'ensemble de tout les orbites périodiques de l'équation différentielle. La seconde partie du $16^{ème}$ problème d'**Hilbert** est reliée au le nombre maximum des cycles limites d'un champ vecteur polynomial ayant un degré fixé. Ce problème et la conjecture de **Riemann** sont les seuls deux problèmes dans la liste d'**Hilbert** qui n'ont pas été résolus. Ici on considère un cas très particulier du $16^{ème}$ problème d'**Hilbert**. On étudie la borne supérieure du système polynomial généralisé de **Kukles**

$$\begin{cases} \dot{x} = -y \\ \dot{y} = Q(x, y), \end{cases} \tag{1}$$

où $Q(x, y)$ est le polynôme avec des coefficients réels du degré n.
Kukles, en 1944 a introduit le système différentiel :

$$\begin{cases} \dot{x} = -y \\ \dot{y} = x + a_1x^2 + a_2xy + a_3y^2 + a_4x^3 \\ \qquad +a_5x^2y + a_6xy^2 + a_7y^3, \end{cases} \tag{2}$$

et a donné la condition nécessaire et suffisante pour que le système ait un centre à l'origine. Ce système cubique sans le terme y^3 est appelé réduit.

Dans [1], **Llibre** et **Mereu** ont étudiés le nombre maximum de cycles limites des systèmes différentielles polynomiales de **Kukles**.

$$\left\{\begin{array}{l} \dot{x}=y \\ \dot{y}=-x-\sum\limits_{k\geq 1}\varepsilon^{k}\left(f_{n}^{k}(x)+g_{m}^{k}(x)\,y+h_{l}^{k}(x)\,y^{2}+d_{0}^{k}y^{3}\right) \end{array}\right. , \tag{3}$$

où pour chaque k les polynômes $f_n^k(x)$, $g_m^k(x)$ et $h_l^k(x)$ ont respectivement les degrés n, m et l, $d_0^k \neq 0$ est un nombre réel et ε est un petit paramètre.

Dans ce travail, on a étudié le nombre maximum de cycles limites donné par la théorie de la moyennisation du premier et seconde ordre, qui peuvent bifurquer des orbites périodiques du centre linéaire

$$\left\{\begin{array}{l} \dot{x}=y \\ \dot{y}=-x, \end{array}\right. \tag{4}$$

perturbé dans la classe des systèmes différentiels polynomiaux généralisés de Kukles suivante :

$$\left\{\begin{array}{l} \dot{x}=y \\ \dot{y}=-x-f(x)-g(x)y-h(x)y^{2}-l(x)y^{3}, \end{array}\right. \tag{5}$$

où $f(x)=\varepsilon f_1(x)+\varepsilon^2 f_2(x), g(x)=\varepsilon g_1(x)+\varepsilon^2 g_2(x), h(x)=\varepsilon h_1(x)+\varepsilon^2 h_2(x)$ et $l(x)=\varepsilon l_1(x)+\varepsilon^2 l_2(x)$ pour chaque $k=1,2$ les polynômes $f_k(x)$,$g_k(x)$
,$h_k(x)$ et $l_k(x)$ ont respectivement les degrés n_1, n_2, n_3 et n_4, et ε est un petit paramètre.
.

Résultats principaux:

Nos résultats sont les suivants :

Theorme 1. le nombre maximum de cycles limites des systèmes différentiels polynomiaux de Kukles (5) bifurquant des orbites périodiques du centre linéaire (4), en utilisant la théorie de moyennisation.

$a)$ cas du premier ordre est :

$$\max\left\{\left[\frac{n_2}{2}\right],\left[\frac{n_4}{2}\right]+1\right\}.$$

$b)$ cas du second ordre est :

$$\max\left\{\left[\frac{n_2}{2}\right],\left[\frac{n_4}{2}\right]+1,\left[\frac{n_1}{2}\right]+\left[\frac{n_2-1}{2}\right],\left[\frac{n_1}{2}\right]+\left[\frac{n_4-1}{2}\right]+1,\right.$$

$$\left[\frac{n_1-1}{2}\right]+\mu, \left[\frac{n_2-1}{2}\right]+\left[\frac{n_3}{2}\right]+1, \left[\frac{n_3}{2}\right]+\left[\frac{n_4-1}{2}\right]+2,$$
$$\left.\left[\frac{n_3-1}{2}\right]+\mu+1\right\},$$

où $\mu=\min\left\{\left[\frac{n_2}{2}\right], \left[\frac{n_4}{2}\right]+1\right\}$ et $[.]$ désigne la fonction partie entière.

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La reduction de R. Smith et application

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Résumé

Russell.A. Smith a développé une méthode de réduction lui permettant de ramener l'étude de certains aspects d'une équation différentielle ordinaire dans $\mathbb{R}^n$ à l'étude de ces mêmes aspects à une équation différentielle ordinaire dans $\mathbb{R}^n$ dite projection de Smith. Dans ce travail nous présentons cette méthode de réduction et donner une application à un système differentielles d'ordre trois.

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On the limit cycles of family of planar polynomial differential systems of degree $2n+1$

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Abstract: In this work we present a family of planar polynomial differential systems of degree $2n+1$ with $n \in \mathbb{N}^*$, then we prove that these systems are integrable and we introduce an explicit expression of a first integral. Moreover, we determine sufficient conditions for the existence of an explicit algebraic or non-algebraic limit cycle.
Keywords: First integral, algebraic and non-algebraic limit cycle, planar polynomial differential system.

Introduction

In the qualitative theory of autonomous and planar differential systems, the study of limit cycles is very attractive because of their relation with the applications to other areas of sciences, see for instance [4]. Nevertheless, most of researchers on that domain focus their attention on the number, stability and location in the phase plane of the limit cycles for the system of degree $n = \max\{\deg P, \deg Q\}$

$$\begin{array}{l} \dot{x} = \frac{dx}{dt} = P(x,y) \\ \dot{y} = \frac{dy}{dt} = Q(x,y) \end{array} \tag{1}$$

Where $P(x,y)$ and $Q(x,y)$ are coprime polynomials of $\mathbb{R}[x,y]$. A limit cycle of system (1) is an isolated periodic solution in the set of all periodic solutions of system (1). If a limit cycle is contained in an algebraic curve of the plane, then we say that it is algebraic, otherwise it is called non-algebraic. In general, it is not easy to distinguish when a limit cycle is algebraic or not, see for example [1,2,3,5]. We recall that an algebraic curve defined by $U(x,y)=0$ is an invariant curve for (1) if there exists a polynomial $K(x,y)$ called the cofactor such that

$$P(x,y)\frac{\partial U}{\partial x}+Q(x,y)\frac{\partial U}{\partial x}=K(x,y)U(x,y).$$

System (1) is integrable on an open set Ω of $\mathbb{R}^2$ if there exists a non constant C^1 function $H:\Omega\to\mathbb{R}$, called a first integral of the system on Ω , which is constant on the trajectories of the system (1) contained in Ω, i.e. if

$$\frac{dH\left(x,y\right)}{dt}=P\left(x,y\right)\frac{\partial H\left(x,y\right)}{\partial x}+Q\frac{\partial H\left(x,y\right)}{\partial y}\equiv 0$$

in the points of Ω. Moreover, $H=h$ is the general solution of this equation, where h is an arbitrary constant. It is well known that for differential systems defined on the plane $\mathbb{R}^2$ the existence of a first integral determines their phase portrait.

In this work we are interested in studying the integrability and the limit cycles of family of polynomial differential system of the form

$$\begin{cases} \dot{x}=bx+\left(ax-y\right)\left(x^2+y^2\right)^n+\left(cx-y\right)R_{2n}\left(x,y\right) \\ \dot{y}=by+\left(ay+x\right)\left(x^2+y^2\right)^n+\left(x+cy\right)R_{2n}\left(x,y\right) \end{cases} \tag{2}$$

where a, b, c are real parameters and $R_{2n}(x,y)$ is a homogeneous polynomial of degree $2n$. System (2) can be written in polar coordinates (r,θ) defined by $x=r\cos\theta$, $y=r\sin\theta$, as

$$\begin{cases} \dot{r}=f\left(\theta\right)r^{2n+1}+br, \\ \dot{\theta}=g\left(\theta\right)r^{2n}, \end{cases} \tag{3}$$

where $f\left(\theta\right)=a+cR_{2n}\left(\cos\theta,\sin\theta\right)$, $g\left(\theta\right)=R_{2n}\left(\cos\theta,\sin\theta\right)+1$.

Main result

As a main result, we shall prove the following theorem.

Theorem:

Consider a polynomial differential system with homogeneous nonlinearity (2). Then the following statements hold.

a) The curve $F\left(x,y\right)=\left(R_{2n}+\left(x^2+y^2\right)^n\right)\left(x^2+y^2\right)$ is an invariant algebraic curve of

system (2).
b) If $g(\theta) \neq 0$ for all $\theta \in [0, 2\pi)$,then system (2) has the first integral

$$I(x,y) = \left(x^2+y^2\right) e^{-2n \int_0^{\arctan \frac{y}{x}} \frac{f(s)}{g(s)} ds} - 2n \int_0^{\arctan \frac{y}{x}} \frac{b}{g(u)} e^{-2n \int_0^u \frac{f(s)}{g(s)} ds} du$$

c) If $g(\theta)$ vanishes for some $\theta \in [0, 2\pi)$, then system (2) has no periodic limit cycles.
d) If $g(\theta) \equiv w+1$ for all $\theta \in [0, 2\pi)$ (w is an constant)and $(a+cw)\, b < 0$, then system (2) has a unique algebraic limit cycle whose expression is

$$\left(x^2+y^2\right)^n + \frac{b}{a+cw} = 0$$

e) If and $R_{2n}(\cos\theta, \sin\theta) > 0$ for all $\theta \in [0, 2\pi)$,and one of the following conditions is satisfied: (i) If $a < 0, c \leq 0, b > 0$
(ii) if $a > 0, c \geq 0, b < 0$
Then system (2) has a unique non algebraic limit cycle whose expression in polar coordinates (r, θ) defined by $x = r\cos\theta$ and $y = r\sin\theta$, is

$$r(\theta, r_*) = e^{\int_0^\theta \frac{f(s)}{g(s)} ds} \left(r_*^{2n} + 2n \int_0^\theta \frac{b}{g(u)} e^{-2n \int_0^u \frac{f(s)}{g(s)} ds} du \right)^{\frac{1}{2n}},$$

Where

$$r_*^{2n} = \frac{2n e^{2n \int_0^{2\pi} \frac{f(s)}{g(s)} ds} \int_0^{2\pi} \frac{b}{g(u)} e^{-2n \int_0^u \frac{f(s)}{g(s)} ds} du}{1 - e^{2n \int_0^{2\pi} \frac{f(s)}{g(s)} ds}}$$

Example
let $a = 2$, $b = -2$, $c = 1$ and $R_2(x,y) = x^2 + xy + y^2$, then system (2) becomes

$$\begin{aligned} \dot{x} &= bx + (ax - y)\left(x^2+y^2\right) + (cx - y)\left(x^2+xy+y^2\right) \\ \dot{y} &= by + (ay + x)\left(x^2+y^2\right) + (x + cy)\left(x^2+xy+y^2\right) \end{aligned} \tag{4}$$

This system has a unique unstable and hyperbolic non-algebraic limit cycle surrounding a stable node at the origin, whose expression in polar coordinates $(r;\theta)$ is

$$r(\theta, r_*) = e^{\int_0^\theta \frac{f(s)}{g(s)} ds} \left(r_*^{2n} - 2 \int_0^\theta \frac{1}{g(u)} e^{-2 \int_0^u \frac{f(s)}{g(s)} ds} du \right)^{\frac{1}{2}}$$

where $f(s) = 3 + \frac{1}{2}\sin 2s$; $\quad g(s) = 2 + \frac{1}{2}\sin 2s$ and $r_* = 0.557\,05$

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Existence and uniqueness of solutions for the nonlinear fractional differential equations with nonlocal conditions

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Abstract: In this work, we use the Banach contraction mapping principle and the Krasnoselskii fixed point theorem to obtain the existence and uniqueness of solutions for nonlinear retarded and advanced implicit Hadamard fractional differential equations with nonlocal conditions.

Keywords: Fractional differential equation, Hadamard fractional derivative, Fixed point theorem, Existence and uniqueness, Nonlocal condition.

Introduction

Fractional delay differential equations (FDDE) are dynamical systems involving non integer order derivatives as well as time delays. These equations have found many applications in control theory, agriculture, chaos, bioengineering, economy, control systems, physics, chemistry, biology, medicine, atomic energy, information theory, harmonic oscillator, nonlinear oscillations, conservative systems, stability and instability of geodesic on Riemannian manifolds, dynamics in Hamiltonian systems, etc. In particular, problems concerning qualitative analysis of linear and nonlinear fractional differential equations with and without delay have received the attention of many authors.

Let $C([-r,h],\mathbb{R})$ be the Banach space of continuous function with the norm

$$\|y\|_{[-r,h]} = \sup\{|y(t)| : -r \le t \le h\}.$$

The spaces $C([1-r,e+h],\mathbb{R})$ of the continuous functions y from $[1-r,e+h]$ into $\mathbb{R}$ with the norm

$$\|y\|_{[1-r,e+h]} = \sup\{|y(t)| : 1-r \le t \le e+h\}.$$

The spaces $L^1(J, \mathbb{R})$ of Lebesgue-integrable functions $w : J \longrightarrow \mathbb{R}$ with the norm

$$\|w\|_1 = \int_1^T |w(s)|\, ds.$$

We are interested in the analysis of qualitative theory of the problems of the existence and uniqueness of solutions to nonlinear retarded and advanced Hadamard fractional differential equations. Inspired and motivated by the work of [1], we concentrate on the existence and uniqueness of solutions for the nonlinear retarded and advanced implicit Hadamard fractional differential equation with nonlocal conditions

$$\begin{cases} D^\alpha y(t) = f(t, y_t, D^\alpha y(t)), \text{ for each, } t \in J := [1, e], 1 < \alpha \leq 2, \\ y(t) + (H_1 y)(t) = \kappa(t), \ t \in [1-r, 1], \ r > 0, \\ y(t) + (H_2 y)(t) = \psi(t), \ t \in [e, e+h], \ h > 0, \end{cases} \tag{1}$$

where D^α is the Hadamard fractional derivative, $f : J \times C([-r, h], \mathbb{R}) \times \mathbb{R} \to \mathbb{R}$ is a given continuous function, $H_1 : C([1-r, e+h], \mathbb{R}) \to C([1-r, 1], \mathbb{R})$ and $H_2 : C([1-r, e+h], \mathbb{R}) \to C([e, e+h], \mathbb{R})$ are given continuous mappings, $\kappa \in C([1-r, 1], \mathbb{R})$ and $\psi \in C([e, e+h], \mathbb{R})$.

For each function y defined on $[1-r, e+h]$ and for any $t \in J$, we denote by y_t the element of $C([-r, h], \mathbb{R})$ defined by

$$y_t(\theta) = y(t+\theta), \theta \in [-r, h].$$

To show the existence and uniqueness of solutions, we transform (1) into an integral equation and then use the Banach contraction mapping principle and the Krasnoselskii fixed point theorem.

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New results on positive bounded solutions of a second-order iterative functional differential equation

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Abstract: The main goal of this work is the investigation of a nonlinear second-order iterative differential equation with integral boundary conditions. With the help of Schauder's fixed point theorem, we establish some sufficient conditions that ensure the existence, uniqueness and continuous dependence of a positive bounded solution for our iterative equation. Our main findings enrich and complement those available in the literature.

Keywords: Schauder's fixed point theorem, Second-order Iterative differential equation, Integral boundary conditions.

1. Introduction

Iterative differential equations can be considered as a particular type of the so-called differential equations with time and state dependent delays. They have gained considerable interest by many authors due to their wide variety of scientific applications. For instance, they can modelize, the infectious disease transmission in epidimyology, population dynamics in ecology and draining or coating fluid flow problems (see for example [4,5,1,2] and references therein). The main task of this work is to establish the existence, uniqueness and continuous dependence of a positive bounded solution for the following second order iterative problem which can describe one-dimensional diffusion phenomena with an iterative source or a reaction term:

$$x''(t) + f\left(t, x^{[1]}(t), ..., x^{[n]}(t)\right) = \frac{d}{dt} g\left(t, x^{[1]}(t), ..., x^{[n]}(t)\right), \ 0 < t < b, \tag{1}$$

$$x(0) = 0, \ \alpha \int_0^\eta x(s)\, ds = x(b) \ \text{ with } \eta \in (0, b), \ \alpha \in \mathbb{R}^*, \tag{2}$$

where $x^{[m]}(t)$ is the $m\ th$ iterate of the function $x(t)$ and $f, g : [0, b] \times \mathbb{R}^n \longrightarrow [0, +\infty)$ are continuous functions with respect to their arguments. Here we would like to point out that due to the existence of the iterative terms, the study of these equations is often very difficult. Our approach is based on the conversion of our problem into a fixed point problem by pursuing the following steps: Firstly, we will choose an appropriate Banach space for making the iterative terms $x^{[2]}(t), ..., x^{[n]}(t)$ well-defined and applying the Schauder's fixed point theorem. Secondly, we will convert our boundary-value problem into an equivalent integral equation. Finally, we will use Arzela-Ascoli theorem and Schauder's fixed point theorem to prove our main results.

2. Preliminaries

We define a subset $\mathcal{CB}_{Int}$ of $\mathcal{C}([0,b], \mathrm{I\!R})$ as follows:

$$\mathcal{CB}_{Int} = \left\{ x \in \mathcal{C}([0,b], \mathrm{I\!R}) : x(0) = 0,\ \alpha \int_0^\eta x(s)\, ds = x(b),\ \alpha \in \mathrm{I\!R}^*,\ \eta \in (0,b) \right\}.$$

It's clear that $(\mathcal{CB}_{Int}, \|.\|)$ is a Banach space. For $0 \leq L \leq b$ and $M \geq 0$, let

$$\mathcal{CB}_{Int}(L,M) = \{x \in \mathcal{CB}_{Int},\ 0 \leq x \leq L,\ \ |x(t_2) - x(t_1)| \leq M |t_2 - t_1|,\ \forall t_1, t_2 \in [0,b]\},$$

then $\mathcal{CB}_{Int}(L,M)$ is a closed convex and bounded subset of $\mathcal{CB}_{Int}$.

Throughout this paper we assume that the functions $f(t, x_1, x_2, ..., x_n)$ and $g(t, x_1, x_2, ..., x_n)$ are globally Lipschitz in $x_1, ..., x_n$. i.e., there exist n positive constants $c_1,\ c_2, ..., c_n$ and n positive constants $k_1, k_2...., k_n$ such that

$$|f(t, x_1, ..., x_n) - f(t, y_1, ..., y_2)| \leq \sum_{i=1}^{n} c_i \|x_i - y_i\|, \tag{3}$$

$$|g(t, x_1, ..., x_n) - g(t, y_1, ..., y_2)| \leq \sum_{i=1}^{n} k_i \|x_i - y_i\| \tag{4}$$

and we introduce the following constants:

$$\rho = \sup_{s \in [0,b]} |f(s, 0, 0, ..., 0)|,\ \zeta = \rho + L \sum_{i=1}^{n} c_i \sum_{j=0}^{j=i-1} M^j.$$

Lemma 1. Let $2b \neq \alpha\eta^2$, then for $f \in \mathcal{C}([0,b], [0,+\infty))$ and $g, \in \mathcal{C}^1([0,b], [0,+\infty))$ the problem (1)-(2) has a unique solution given by

$$x(t) = 2t\int_0^b \frac{(b-s)}{2b-\alpha\eta^2} f\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds - \alpha t\int_0^\eta \frac{(\eta-s)^2}{2b-\alpha\eta^2} f\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds$$
$$- \int_0^t (t-s) f\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds - \frac{2t}{2b-\alpha\eta^2}\int_0^b g\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds$$
$$+ \frac{2\alpha t}{2b-\alpha\eta^2}\int_0^\eta (\eta-s) g\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds + \int_0^t g\left(s, x^{[1]}(s), ..., x^{[n]}(s)\right) ds.$$

3. Main results

Existence

By virtue of Lemma 1, we define an operator $\mathcal{A} : \mathcal{CB}_{Int}(L, M) \longrightarrow \mathcal{CB}_{Int}$ as follows:

$$(\mathcal{A}\varphi)(t) = 2t\int_0^b \frac{(b-s)}{2b-\alpha\eta^2} f\left(s, \varphi^{[1]}(s), ..., \varphi^{[n]}(s)\right) ds - \alpha t\int_0^\eta \frac{(\eta-s)^2}{2b-\alpha\eta^2} f\left(s, \varphi^{[1]}(s), ..., \varphi^{[n]}(s)\right) ds$$
$$- \int_0^t (t-s) f\left(s, \varphi^{[1]}(s), ..., \varphi^{[n]}(s)\right) ds - \frac{2t}{2b-\alpha\eta^2}\int_0^b g\left(s, \varphi^{[1]}(s), ..., \varphi^{[n]}(s)\right) ds$$
$$+ \frac{2\alpha t}{2b-\alpha\eta^2}\int_0^\eta (\eta-s) g\left(s, \varphi^{[1]}(s), ..., \varphi(s)\right) ds + \int_0^t g\left(s, \varphi^{[1]}(s), ..., \varphi^{[n]}(s)\right) ds. \tag{5}$$

φ is a solution of the boundary-value problem (1)-(2) if and only if φ is a fixed point of the operator $\mathcal{A}$. By virtue of the Arzelà-Ascoli theorem, we can show that the closed subset $\mathcal{CB}_{Int}(L, M)$ of $\mathcal{CB}_{Int}$. So, for proving the existence of solutions of (1)-(2), it suffices to show that $\mathcal{A}$ is well defined, continuous and $A(\mathcal{CB}_{Int}(L, M)) \subset \mathcal{CB}_{Int}(L, M)$.

Lemma 2. Let $2b \neq \alpha\eta^2$, then operator A given by (5) is well defined.

Lemma 3. Suppose that condition (3)and (4)holds. Then the operator $\mathcal{A}$ given by (5) is continuous.

Lemma 4. Suppose that condition (3),(4) holds. If

$$b\zeta\left(\frac{3b^2 + |\alpha|\eta^3}{3|2b-\alpha\eta^2|} + \frac{1}{2}b\right) + \frac{4b}{|2b-\alpha\eta^2|}\omega \leq L, \tag{6}$$

and

$$\left(\frac{\zeta(3b^2 + \eta^3|\alpha|)}{3|2b-\alpha\eta^2|} + \zeta b + \frac{4b\omega}{|2b-\alpha\eta^2|}\right) \leq M, \tag{7}$$

then $A\mathcal{CB}_{Int}(L, M)) \subset \mathcal{CB}_{Int}(L, M)$.

Theorem 1. Suppose that conditions (3), (4), (6) and (7) hold. Then the problem (1)-(2)

has at least one positive bounded solution x in $\mathcal{CB}_{Int}(L,M)$.

Uniqueness

Theorem 2. Under the hypotheses of Theorem 1, assume further that

$$\left(\left(b\left(\frac{3b^2+|\alpha|\eta^3}{3|2b-\alpha\eta^2|}+\frac{1}{2}b\right)\sum_{i=1}^{n}c_i\sum_{j=0}^{j=i-1}M^j\right)+\left(\frac{4b^2}{|2b-\alpha\eta^2|}\sum_{i=1}^{n}k_i\sum_{j=0}^{j=i-1}M^j\right)\right)<1, \qquad (8)$$

then problem (1)-(2) has a unique solution in $\mathcal{CB}_{Int}(L,M)$.

Continuous dependence

Theorem 3. Suppose that the conditions of Theorem 2 hold. The unique solution of (1)-(2) depends continuously on the functions f and g.

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On the stability of certain nonlinear delay dynamic equations

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Abstract: The theory of time scale calculus can be applicable to any field in which dynamic processes are described by discrete or continuous time models. The aim of the present work is to use some of those results and to apply Krasnoselskii fixed point theorem to obtain stability results about the zero solution for nonlinear delay dynamic equations..
Keywords: Delay differential equations, fixed point, stability, time scales.

Introduction

The theory of fixed point is one of the most powerful tools of modern mathematics. Theorem concerning the existence and properties of fixed points are known as fixed point theorem. Fixed point theory is a beautiful mixture of analysis, topology and geometry. In particular fixed point theorem has been applied in such field as mathematics engineering, physics, economics, game theory, biology and chemistry etc. Classical and major results in these areas are: Banach's fixed point theorem, Schauder's fixed point theorem and Krasnoselskii's fixed point theorem. Time scale theory was introduced for the first time by Stefan Hilger in 1988 to unify continuous and discrete analysis. However, some physical systems are modeled by so-called dynamic equations because they are differential equations, difference equations or a combination of both. Therefore, the calculation of time scales provides a generalization of the differential analysis and the difference. The study of delay dynamic equations has become important applications, for mathematical models as well as in physics, population dynamics and economics, there has been much research activity concerning the various equations on time scales. The aim of this work is to use Krasnoselskii's fixed point theorem to obtain stability results about the zero solution for following nonlinear delay dynamic equations

$$x^{\Delta}(t)+\int_{t-\tau(t)}^{t} a(t,s)g\left(x(s)\right)\Delta s+c(t)x^{\widetilde{\Delta}}(t-\tau(t))=0,\ t\in[t_0,\infty)_{\mathbf{T}}, \tag{1}$$

with an assumed initial condition

$$x\left(t\right)=\phi\left(t\right),\ t\in\left[m\left(t_0\right),t_0\right]_{\mathbf{T}},$$

where $\phi\in C_{rd}\left(\left[m\left(t_0\right),t_0\right]_{\mathbf{T}},\mathrm{I\!R}\right)$ and

$$m\left(t_0\right)=\inf\left\{t-\tau\left(t\right):t\in\left[t_0,\infty\right)_{\mathbf{T}}\right\}.$$

Throughout this paper, we assume that $c\in C_{rd}^{1}\left([t_0,\infty)_{\mathbf{T}},\mathrm{I\!R}\right)$, $a\in C_{rd}([t_0,\infty)_{\mathbf{T}}\times[m\left(t_0\right),\infty)_{\mathbf{T}},\mathrm{I\!R}_{+})$ and $g:\mathrm{I\!R}\to\mathrm{I\!R}$ is continuous with respect to its argument. We assume that $g\left(0\right)=0$ and $\tau\in C_{rd}^{2}\left([t_0,\infty)_{\mathbf{T}},\mathrm{I\!R}^{+}\right)$ such that

$$\tau^{\Delta}\left(t\right)\neq 1,\ t\in[t_0,\infty)_{\mathbf{T}}. \tag{2}$$

Our purpose here is to use the Krasnoselskii-Burton's fixed point theorem to show the asymptotic stability and stability of the zero solution for (1).

Main results

One crucial step in the investigation of an equation using fixed point theory involves the construction of a suitable fixed point mapping. For that end we must invert (1) to obtain an equivalent integral equation from which we derive the needed mapping. During the process, an integration by parts has to be performed on the neutral term $x^{\widetilde{\Delta}}(t-\tau(t))$. Unfortunately, when doing this, a derivative $\tau^{\Delta}(t)$ of the delay appears on the way, and so we have to support it.

Lemma. Suppose that (2) holds. Then x is a solution of equation (1) if and only if

$$\begin{aligned} x(t) &= \left(\phi(t_0)+\gamma(t_0)\phi(t_0-\tau(t_0))\right)e_{\ominus A}\left(t,t_0\right) \\ &\quad+\int_{t_0}^{t}\left(\int_{s-\tau(s)}^{s} a(s,u)\left(Gx\right)(u)du\right)e_{\ominus A}\left(t,s\right)\Delta s-\gamma(t)x(t-\tau(t)) \\ &\quad-\int_{t_0}^{t}\left[L_x(s)-\varrho(s)x^{\sigma}(s-\tau(s))\right]e_{\ominus A}\left(t,s\right)\Delta s,\ t\in\ [t_0,\infty)_{\mathbf{T}}, \end{aligned} \tag{3}$$

where

$$L_x(t) = \int_{t-\tau(t)}^{t} a(t,s) \left(\int_{s}^{\sigma(t)} \left(\int_{u-\tau(u)}^{u} a(u,v)x(v)dv - r(u)x^{\sigma}(u-\tau(u)) \right) \Delta u \right.$$
$$\left. + \gamma^{\sigma}(t)\, x(\sigma(t) - \tau^{\sigma}(t)) - \gamma(s)\, x(s - \tau(s)) \right) \Delta s \tag{4}$$

$$r(t) = \frac{c^{\Delta}(t)(1-\tau^{\Delta}(t)) + \tau^{\Delta\Delta}(t)c(t)}{(1-\tau^{\Delta}(t))\,(1-\tau^{\Delta}(\sigma(t)))},\ \gamma(t) = \frac{c(t)}{1-\tau^{\Delta}(t)}, \tag{5}$$

$$(Gx)(t) = x(t) - g(x(t)), \tag{6}$$

and

$$\varrho(t) = \frac{(c^{\Delta}(t) + c^{\sigma}(t)A(t))(1-\tau^{\Delta}(t)) + \tau^{\Delta\Delta}(t)c(t)}{(1-\tau^{\Delta}(t))\,(1-\tau^{\Delta}(\sigma(t)))},\ A(t) = \int_{t-\tau(t)}^{t} a(t,s)\Delta s. \tag{7}$$

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Pseudo almost periodic solution for a delayed Nicholson's blowflies model with Stepanov pseudo almost periodic coefficients

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Abstract: In this work, we prove the existence and uniqueness of pseudo-almost periodic solutions for a delayed Nicholson's blowflies model with Stepanov pseudo almost periodic coefficients and harvesting term. The results are obtained by using the Banach fixed point theorem and the properties of the Stepanov pseudo-almost periodic functions.

Keywords: Stepanov pseudo-almost periodic functions, Bochner transform, Nicholson blowflies model, Banach fixed point theorem.

Introduction

To describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained by Nicholson [7], Gurney et al.[6] proposed the following delay differential equation, known as the famous Nicholson blowflies model:

$$\dot{x}(t) = -\delta x(t) + px(t-\tau)e^{ax(t-\tau)}, \tag{1}$$

where $x(t)$ is the size of the population at time t, p is the maximum per capita daily egg production rate, $\frac{1}{a}$ is the size at which the blowfly population reproduces at its maximum rate, δ is the per capita daily adult death rate, τ is is the generation rate and t is the generation time.

Nicholson blowflies models (1) have been naturally generalized, in particular to variable coefficients and delays, in order to describe population dynamics in natural environments. The study of population dynamics with harvesting has been also discussed by several authors. Especially, Berezansky et al. [2] proposed the following Nicholson's blowflies model with linear harvesting term:

$$\dot{x}(t) = -\delta x(t) + p(t)x(t-\tau)e^{a(t)x(t-\tau)} - H(t)x(t-\sigma), \tag{2}$$

where $p(.), a(.), H(.)$ are functions.

The initial condition of model (2) takes the form

$$x_t(.) = \phi(.),$$

where $\phi \in C([-r, 0], \mathbb{R}^+)$ *and* $\phi(0) > 0,\ r = \max\{\tau, \sigma\}$.

In recent years, there has been considerable interest in the study of the qualitative properties of (1) and its variants, for instance, periodicity, almost periodicity, pseudo-almost periodicity, oscillation and stability, we can cite for example the works [3,5,6] and the references cited therein.

Duan et al [5], considered the equation (2) when the functions $p(.), a(.), H(.)$ are pseudo almost periodic functions. Motivated by the results of [5], the main purpose of this paper is to give sufficient conditions for the existence and uniqueness of positive pseudo almost periodic solutions for the model (2) when the coefficients are Stepanov-pseudo almost periodic.

1. Stepanov pseudo-almost periodic functions

In the early nineties, Zhang [8] introduced a significant generalization of almost periodic functions, the so called pseudo almost periodic functions by disturbing the almost periodic function by an ergodic term. Namely: A function $f \in BC(\mathbb{R}, \mathbb{R})$ is called pseudo-almost periodic ($f \in PAP\,(\mathbb{R}, \mathbb{R})$) if $f = g + \varphi$ with $g \in AP(\mathbb{R}, \mathbb{R})$ and $\varphi \in \mathcal{E}(\mathbb{R}, \mathbb{R})$ where

$$\mathcal{E}(\mathbb{R}, \mathbb{R}) = \{\varphi \in BC(\mathbb{R}, \mathbb{R}),\ \lim_{T \to +\infty} \frac{1}{2T} \int_{-T}^{T} |\varphi(t)|\, dt = 0\},$$

and $g \in AP\,(\mathbb{R}, \mathbb{R})$ means that g is continuous and for all $\varepsilon > 0$ the set

$$T(f, \varepsilon) := \{\tau \in \mathbb{R}, \quad \|f_\tau - f\|_\infty < \varepsilon\},$$

is relatively dense in $\mathbb{R}$ (see [1]).

Definition: [4] A function $f \in BS^p\,(\mathbb{R}, \mathbb{R}),\ 1 \leq p < \infty$, is called Stepanov pseudo-almost periodic or S^p-pseudo-almost periodic (we write $f \in PAPS^p(\mathbb{R})$) if it can be decomposed as

$$f = g + h.$$

with $g^b \in AP(\mathbb{R}, L^p([0, 1]; \mathbb{R}))$ and $h^b \in \mathcal{E}(\mathbb{R}, L^p([0, 1], \mathbb{R}))$.

Recall that

$$BS^{p}(\mathbb{R},\mathbb{R}) := \left\{ f \in M(\mathbb{R},\mathbb{R}) : \ \|f\|_{S^p} < \infty \right\},$$

where $\|f\|_{S^p} = \|f^b\|_{L^\infty(\mathbb{R},(L^p[0,1],\mathbb{R}))} = \sup\limits_{t\in\mathbb{R}} \left(\int_t^{t+1} |f(\tau)|^p \, d\tau \right)^{\frac{1}{p}}$, and f^b denotes the Bochner transform of f defined as follows

$$f^b(t) = f(t+s), \ \forall s \in [0,1], \ \forall t \in \mathbb{R}.$$

2. Main results

We make the following assumptions on the equation (2):

1. $a(.)$ is pseudo-almost periodic functions from $\mathbb{R}^+$ to $\mathbb{R}$.

2. $p(.), \ H(.)$ are Stepanov pseudo-almost periodic functions from $\mathbb{R}^+$ to $\mathbb{R}$.

Theorem. If the conditions (1), (2) are satisfied, and we have

1. for $1 < p < +\infty$

$$r = \frac{e^{-\delta}}{(1-e^{-\delta})} \Big(\frac{e^q - 1}{\delta q} \Big)^{\frac{1}{q}} \Big(\frac{1}{e^2} ||p||_{S^p} + ||H||_{S^p} \Big) < 1, \tag{3}$$

2. for $p = 1$

$$r = \Big(\frac{1}{e^2} \frac{1}{1-e^{-\delta}} ||p||_{S^1} - \frac{1}{1-e^{-\delta}} ||H||_{S^1} \Big) < 1. \tag{4}$$

then the equation (2) admits an unique pseudo-almost periodic solution in the region

$$\mathbb{B} = \{ x \in PAP(\mathbb{R}^+, \mathbb{R}), \ R_1 \le x(t) \le R_2 \},$$

where R_1 and R_2 are positives constants depending on the functions $a(.), p(.), \ and \ H(.)$

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Existence of positive solutions for dynamic equations on time scales

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Abstract: In this work we study the existence of positive solutions for a dynamic equations on time scales. The main tool employed here is the Schauder's fixed point theorem. The results obtained here extend the work of Olach. Two examples are also given to illustrate this work.

Keywords: Positive solutions, Schauderś fixed point theorem, dynamic equations, time scales.

Introduction

Yong Zhou in [4] studying the existence of nonoscillatory solutions of the second-order nonlinear neutral differential equation

$$\left[r(t)\left(x\left(t\right)-P\left(t\right)x\left(t-\tau\right)\right)'\right]'+\sum_{i=1}^{m}Q_i\left(t\right)f_i\left(x\left(t-\tau_i\right)\right)=0,\ t\geq t_0.$$

I. Culakova, L'. Hanustiakova, Rudolf Olach, in [**?**] studying Existence for positive solutions of second-order neutral nonlinear differential equations

$$\left[r(t)\left(x\left(t\right)-P\left(t\right)x\left(t-\tau_1\right)\right)'\right]'+Q\left(t\right)f\left(x\left(t-\tau_2\right)\right)=0,\ t\geq t_0.$$

In this work, we are interested in the analysis of qualitative theory of solutions of delay dynamic equations. Motivated by the papers [3] and the references therein. Let $\mathbf{T}$ be a time scale such that $t_0\in\mathbf{T}$, we consider the following delay dynamic equation

$$\left[r(t)\left(x\left(t\right)-P\left(t\right)x\left(t-\tau_1\right)\right)^{\Delta}\right]^{\Delta}+Q\left(t\right)f\left(x\left(t-\tau_2\right)\right)=0,\ t\geq t_0. \tag{1}$$

Throughout this work we assume that

(i) $\tau_1 > 0, \tau_2 > 0$, for all $t \geq t_0$, $t - \tau_1 \in \mathbf{T}$ and $t - \tau_2 \in \mathbf{T}$,

(ii) $r, P \in C_{rd}([t_0, \infty) \cap \mathbf{T}, (0, \infty))$, $Q \in C_{rd}(\mathbf{T}, (0, \infty))$, f continuous nondecreasing function and $xf(x) > 0$, $x \neq 0$.

The results presented in this work extend the main results in [3].

Main results

Lemma. Suppose (i) and (ii) hold. Then x is a solution of Eq.(1) if and only if

$$x(t) = P(t)\, x(t - \tau_1) - \int_t^\infty \frac{1}{r(s)} \int_s^\infty Q(\xi) f\left(x(\xi - \tau_2)\right) \Delta\xi \Delta s,$$

where $x(t) - P(t)\, x(t - \tau_1) \to 0$ and $r(t)\left(x(t) - P(t)\, x(t - \tau_1)\right)^\Delta \to 0$.

Theorem. Suppose that $0 < k_1 < k_2$, $\int_{t_0}^\infty \xi_{\mu(v)}(k_1 Q(v))\Delta v = \infty$,and there exist $\gamma \geq 0$ such that $t_0 - \gamma \in \mathbf{T}$,

$$\frac{\ominus(k_1 Q(t))}{\ominus(k_2 Q(t))} e_{(k_2 Q)\ominus(k_1 Q)}(t_0, t_0 - \gamma) \geq 1, \tag{2}$$

$$\begin{aligned} &e_{\ominus(k_2 Q)}(t, t - \tau_1) + e_{(k_2 Q)}(t - \tau_1, t_0 - \gamma) \int_t^\infty \frac{1}{r(s)} \int_s^\infty Q(\xi) \\ &\times f\left(e_{\ominus(k_1 Q)}(\xi - \tau_2, t_0 - \gamma)\right) \Delta\xi \Delta s \leq P(t) \leq e_{\ominus(k_1 Q)}(t, t - \tau_1) \\ &+ e_{(k_1 Q)}(t - \tau_1, t_0 - \gamma) \int_t^\infty \frac{1}{r(s)} \int_s^\infty Q(\xi) f\left(e_{\ominus(k_2 Q)}(\xi - \tau_2, t_0 - \gamma)\right) \Delta\xi \Delta s,\ t \geq t_0. \end{aligned}$$

Then $Eq(1)$ has a positive solution which tends to zero.

Corollary. Suppose that $k > 0$, (2) holds and

$$P(t) = e_{\ominus(kQ)}(t, t - \tau_1) + e_{(kQ)}(t - \tau_1, t_0) \int_t^\infty \frac{1}{r(s)} \int_s^\infty Q(\xi) f\left(e_{\ominus(kQ)}(\xi - \tau_2, t_0)\right) \Delta\xi \Delta s,\ t \geq t_0.$$

Then Eq. (1) has a solution

$$x(t) = e_{\ominus(kQ)}(t, t_0),\ t \geq t_0.$$

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Nonlocal Integro-Differential Boundary Value Problem for Fractional Differential Equation on An Infinite Interval

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Résumé : Dans ce sujet, nous nous intéressons à lexistence de solutions aux équations différentielles fractionnaires soumises aux conditions aux limites intégrales fractionnaires de Riemann-Liouville. Au moyen dun théorme à point fixe, des conditions suffisantes sont obtenues pour garantir lexistence dau moins une solution. un exemple illustre lapplicabilité de notre résultat principal.
Mots clés: Problme aux limites; Intervalle infini; Equation différentielle fractionnaire; Théorme du point fixe.

Introduction

L'objectif de ce sujet est de présenter un résultat de l'existence de solutions pour un problme aux limites d'ordre fractionnaire où la dérivation est du type Riemann-Liouville et o les conditions aux limites sont du type intégral avec dérivée d'ordre fractionnaire et poséent sur un intervalle non borné de la droite réelle. Cet problme est comme suit:

$$\begin{cases} D_{0^+}^{\alpha}u(t) + f(t, u(t), D_{0^+}^{\alpha-1}u(t)) = 0, \qquad t \in (0, +\infty), \\ u(0) = 0, \qquad \lim\limits_{t\to+\infty} D_{0^+}^{\alpha-1}u(t) = \beta I_{0^+}^{\alpha-1}u(\eta), \end{cases} \tag{1}$$

où $1 < \alpha \leq 2$, $\eta > 0$ et $\beta > 0$ satisfaits $0 < \beta\eta^{2\alpha-2} < \Gamma(2\alpha-1)$. $D_{0^+}^{\alpha}$ est la standard dérivée fractionnaire de Riemann-Liouville et $I_{0^+}^{\alpha}$ est la standard intégrale fractionnaire de Riemann-Liouville.

Les travaux présentés à ce sujet s'inscrivent dans la continuité des travaux antérieurs et portent sur les problmes aux limites associé à des équations différentielle à dérivée fractionnaire. Il est principalement motivé par des documents [2, 3, 4, 5]. Pour surmonter la difficulté liée à la compacité de l'opérateur de point fixe, un espace Banach spécial est introduit. Nos résultats permettent à la condition intégrale de dépendre de l'intégrale fractionnaire $I_{0^+}^{\alpha-1}u$ ce qui entrane des difficultés supplémentaires.

1. Lemmes auxiliaires

On définit les espaces de Banach X et Y par

$$X = \left\{ u \in C([0, +\infty), \mathbb{R}) : \quad \sup_{t\geq 0} \frac{|u(t)|}{1+t^\alpha} < +\infty \right\}$$

muni de la norme

$$\|u\|_X = \sup_{t\geq 0} \frac{|u(t)|}{1+t^\alpha}$$

$$\begin{aligned} Y \;=\; & \Big\{ u \in X, \quad D_{0^+}^{\alpha-1}u \text{ existe}, \quad D_{0^+}^{\alpha-1}u \in C([0, +\infty), \mathbb{R}), \\ & \sup_{t\geq 0} |D_{0^+}^{\alpha-1}u(t)| < +\infty \Big\} \end{aligned}$$

avec la norme

$$\|u\|_Y = \max \left\{ \sup_{t\geq 0} \frac{|u(t)|}{1+t^\alpha}, \quad \sup_{t\geq 0} |D_{0^+}^{\alpha-1}u(t)| \right\}.$$

Tout d'abord, nous listons quelques hypothèses:

$(H1)$ $0 < \beta\eta^{2\alpha-2} < \Gamma(2\alpha - 1)$.

$(H2)$ La fonction $f : [0, +\infty)\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ est continue telle que $\int_0^{+\infty} |f(s, 0, 0)|ds < +\infty$.

$(H3)$ Il existe deux fonctions positives $(1 + t^\alpha)g(t),\ h(t) \in L^1[0, +\infty)$ telles que $|f(t, x, \overline{x}) - f(t, y, \overline{y})| \leq g(t)|x - y| + h(t)|\overline{x} - \overline{y}|$ pour tous $x,\ y,\ \overline{x},\ \overline{y} \in \mathbb{R}$ et $t \in [0, +\infty)$.

$(H4)$ $\frac{\beta\eta^{2\alpha-2}}{\Gamma(2\alpha-1)-\beta\eta^{2\alpha-2}} \int_0^{+\infty} ((1 + s^\alpha) g(s) + h(s)) ds < \Gamma(\alpha)$.

$(H5)$ Il existe $\rho > 0$ telle que

$\frac{\rho\big(\Gamma(2\alpha-1)-\beta\eta^{2\alpha-2}\big)}{\Gamma(2\alpha-1)\big(\rho\int_0^{+\infty}((1+s^\alpha)g(s)+h(s))ds+\int_0^{+\infty}|f(s,0,0)|ds\big)} > \frac{1}{\Gamma(\alpha)}$.

Lemma 1. Soit $e(t) \in L^1[0, +\infty)$. Sous hypothèse $(H1)$, le problème

$$\begin{cases} D_{0^+}^{\alpha}u(t) + e(t) = 0, & t \in (0, +\infty), \\ u(0) = 0, \quad \lim_{t\to+\infty} D_{0^+}^{\alpha-1}u(t) = \beta I_{0^+}^{\alpha-1}u(\eta), \end{cases} \tag{2}$$

admet une solution unique donnée par

$$\begin{aligned}u(t) &= -\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}e(s)ds\\ &+\frac{\Gamma(2\alpha-1)t^{\alpha-1}}{\Gamma(\alpha)\left(\Gamma(2\alpha-1)-\beta\eta^{2\alpha-2}\right)}\int_0^{+\infty} e(s)ds\\ &-\frac{\beta t^{\alpha-1}}{\Gamma(\alpha)\left(\Gamma(2\alpha-1)-\beta\eta^{2\alpha-2}\right)}\int_0^{\eta}(\eta-s)^{2\alpha-2}e(s)ds.\end{aligned}$$

Lemma 2. Sous l'hypothèse $(H1)$, la solution de problème (2) peut sécrire

$$u(t)=\int_0^{+\infty} G(t,s)e(s)ds, \text{ où}$$

$$G(t,s)=G_1(t,s)+G_2(t,s),$$

$$G_1(t,s)=\frac{1}{\Gamma(\alpha)}\begin{cases} t^{\alpha-1}-(t-s)^{\alpha-1}, & 0\le s\le t<+\infty,\\ t^{\alpha-1}, & 0\le t\le s<+\infty,\end{cases}$$

$$G_2(t,s)=A\times\frac{1}{\Gamma(\alpha)}\begin{cases} \eta^{2\alpha-2}-(\eta-s)^{2\alpha-2}, & 0\le s\le \eta<+\infty,\\ \eta^{2\alpha-2}, & 0\le \eta\le s<+\infty\end{cases}$$

avec $A=\frac{\beta t^{\alpha-1}}{\Gamma(2\alpha-1)-\beta\eta^{2\alpha-2}}$.

Maintenant, on définit les opérateurs $T_1,\ T_2,\ T$ par

$$\begin{aligned}(T_1u)(t) &= \int_0^{+\infty} G_1(t,s)f(s,u(s),D_{0^+}^{\alpha-1}u(s))ds,\\ (T_2u)(t) &= \int_0^{+\infty} G_2(t,s)f(s,u(s),D_{0^+}^{\alpha-1}u(s))ds,\\ (Tu)(t) &= (T_1u)(t)+(T_2u)(t).\end{aligned}$$

Résoudre le problème (1) revient à la recherche de point fixe pour l'opérateur T.

À présent, nous vérifions que l'opérateur T satisfait toutes les conditions de l'alternative non linéaire de Krasnosel'skii [1].

Lemma 3. [4 (critère de compacité)]

Soit $Z \subseteq Y$ être un ensemble borné. Alors Z est relativement compact on Y si pour tout $u \in Z$, $\frac{u(t)}{1+t^\alpha}$ et $D_{0^+}^{\alpha-1}u(t)$ sont équicontinues sur tous les intervalles compacts de $[0,+\infty)$ et sont équiconvergent à l'infini.

Lemma 4. Soit $\Omega_r = \{u \in Y : \quad \|u\|_Y < r\}$, $(r > 0)$ la boule ouverte de rayon r dans Y.

Lemma 4. Si $(H1)-(H3)$ satisfaits, alors l'ensemble $T(\overline{\Omega}_r)$ est borné.

Lemma 5. Si $(H1)-(H3)$ satisfaits, alors $T_1 : \overline{\Omega}_r \to Y$ est complètement continu.

Lemma 6. Si $(H1)-(H4)$ satisfaits, alors $T_2 : \overline{\Omega}_r \to Y$ est une contraction.

2. Résultat principal

Theorem 1. Résultat principal

Supposons que $(H1)-(H5)$ sont satisfaites. Alors le problème (1) admet au moins une solution.

Exemple: Considérons sur l'intervalle infini le problème aux limites suivant:

$$\begin{cases} D_{0^+}^{\frac{3}{2}}u(t) + \frac{u(t)}{(28+t)^2\left(1+\sqrt{t^3}\right)} + \frac{D_{0^+}^{\frac{1}{2}}u(t)}{3e^t-1} + e^{-t} = 0, \qquad t > 0, \\ \\ u(0) = 0, \qquad \lim\limits_{t\to+\infty} D_{0^+}^{\frac{1}{2}}u(t) = \frac{1}{2}I_{0^+}^{\frac{1}{2}}u(1). \end{cases} \tag{3}$$

Dans ce cas, $\alpha = \frac{3}{2}$, $\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$, $\Gamma(2\alpha-1) = \Gamma(2) = 1$, $\beta = \frac{1}{2}$, $\eta = 1$. Soit

$$f(t,x,y) = \frac{x}{(28+t)^2\left(1+\sqrt{t^3}\right)} + \frac{y}{3e^t-1} + e^{-t}$$

et

$$g(t) = \frac{1}{(28+t)^2\left(1+\sqrt{t^3}\right)}, \quad h(t) = \frac{1}{3e^t-1}.$$

Choisissons

$$\rho > \frac{1}{\frac{7\sqrt{\pi}-1}{28} - \ln(\frac{3}{2})}.$$

On a

$$
\begin{aligned}
\int_0^{+\infty} \left(1+s^{\frac{3}{2}}\right) g(s)ds &= \frac{1}{28} < +\infty, \\
\int_0^{+\infty} h(s)ds = \ln\left(\frac{3}{2}\right) &< +\infty, \\
\int_0^{+\infty} |f(s,0,0)|ds &= \int_0^{+\infty} e^{-s}ds = 1 < +\infty.
\end{aligned}
$$

Il est facile de s'assurer que toutes les conditions du théorème 3 sont satisfaites. Donc, le problème (3) admet une solution.

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Fast growing and fixed points of solutions of complex linear differential equations

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Abstract: The determination of the order of complex functions plays an important role in the study of properties of solutions of complex differential equations. In this presentation, we use a more general concept of order, called the φ-order, to investigate the growth and the oscillation of fixed points of solutions to higher order linear differential equations with analytic coefficients in the unit disc.

Keywords: analytic function, φ-order, φ-type, φ-convergence of exponent, linear differential equation.

Introduction and main results

Let $k \geq 2$, we consider the following differential equation

$$f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_0(z)f = 0, \tag{1}$$

where the coefficients A_0, ..., A_{k-1} are analytic functions in the unit disc

$$\Delta = \{z \in \mathbb{C}, |z| < 1\}.$$

Many authors have investigated the growth of solutions of equation 1 by considering the concept of iterated order (see [1]). This concept do not cover an arbitrary growth of fast growing functions and Semochko [5] has applied a more generalized concept of order called the φ-order. She obtained a precise estimates for the growth of solution f when the coefficient A_0 dominates the growth of coefficients.

Definition 1. [5]

Let φ be an increasing unbounded function on $(0, +\infty)$. The φ-orders of an analytic function f in Δ are defined by

$$\sigma_\varphi^0(f) = \limsup_{r\longrightarrow 1^-} \frac{\varphi\left(M(r,f)\right)}{-\log(1-r)}, \qquad \sigma_\varphi^1(f) = \limsup_{r\longrightarrow 1^-} \frac{\varphi\left(\log^+ M(r,f)\right)}{-\log(1-r)}.$$

where $M(r, f)$ is the maximum modulus of f. If we replace the $\limsup$ by the $\liminf$, we obtain the lower φ-orders of f, $\mu_\varphi^0(f)$ and $\mu_\varphi^1(f)$.

Let Φ denotes the class of positive unbounded increasing functions on $(0, +\infty)$, such that $\varphi(e^t)$ grows slowly, i.e., $\forall c > 0 : \lim\limits_{t \to +\infty} \frac{\varphi(e^{ct})}{\varphi(e^t)} = 1$. For instance, $\log\log(.) \in \Phi$ while $\log(.) \notin \Phi$.

Theorem 1. [5]
Let $\varphi \in \Phi$ and let $A_0, A_1, \ldots, A_{k-1}$ be analytic functions in Δ satisfying

$$\max\left\{\sigma_\varphi^0(A_j), j = 1, \ldots, k-1\right\} < \sigma_\varphi^0(A_0).$$

Then, every solution $f \not\equiv 0$ of 1 satisfies $\sigma_\varphi^1(f) = \sigma_\varphi^0(A_0)$.

In this talk, we present some improvements of the above theorem for the lower φ-order when there are more than one dominant coefficient. We discuss also the oscillation of fixed points of solutions of 1 by considering the φ-convergence exponent.

Theorem 2.
Let $\varphi \in \Phi$ and let $A_0, \ldots, A_{k-1}$ be analytic functions in Δ satisfying

$$\max\left\{\sigma_\varphi^0(A_j) : j = 1, \ldots, k-1\right\} < \mu_\varphi^0(A_0) \leq \sigma_\varphi^0(A_0) < +\infty.$$

Then, every solution $f \not\equiv 0$ of 1 satisfies

$$\mu_\varphi^0(A_0) = \mu_\varphi^1(f) \leq \sigma_\varphi^1(f) = \sigma_\varphi^0(A_0).$$

Definition 2. [3]
Let φ be an increasing unbounded function on $(0, +\infty.)$ The φ-types of an analytic function f in Δ with $0 < \sigma_\varphi^i(f) < +\infty,\ (i = 0, 1)$ are defined by

$$\tau_\varphi^0(f) = \limsup_{r \longrightarrow 1^-} (1-r)^{\sigma_\varphi^0(f)} \exp\left\{\varphi\left(M(r, f)\right)\right\},$$

$$\tau_\varphi^1(f) = \limsup_{r \longrightarrow 1^-} (1-r)^{\sigma_\varphi^1(f)} \exp\left\{\varphi\left(\log^+ M(r, f)\right)\right\}.$$

If we replace the lim sup by the lim inf, we obtain the lower φ-types of f, $\underline{\tau}_{\varphi}^{0}(f)$ and $\underline{\tau}_{\varphi}^{1}(f)$.

Theorem 3.
Let $\varphi \in \Phi$ and let $A_0, \ldots, A_{k-1}$ be analytic functions in Δ. Assume that

$$\max\left\{\sigma_{\varphi}^{0}(A_j) : j = 1, \ldots, k-1\right\} \leq \sigma_{\varphi}^{0}(A_0) = \sigma_0, \ (0 < \sigma_0 < +\infty)$$

and

$$\max\left\{\tau_{\varphi}^{0}(A_j) : \sigma_{\varphi}^{0}(A_j) = \sigma_{\varphi}^{0}(A_0), j \neq 0\right\} < \tau_{\varphi}^{0}(A_0) = \tau_0, \ (0 < \tau_0 < +\infty).$$

Then, every solution $f \not\equiv 0$ of 1 satisfies $\sigma_{\varphi}^{1}(f) = \sigma_{\varphi}^{0}(A_0)$.

Definition 3.
Let φ be an increasing unbounded function on $(0, +\infty)$. The φ-convergence exponents of the sequence of zeros of a meromorphic function f in Δ are defined by

$$\lambda_{\varphi}^{0}(f) = \limsup_{r \to 1^-} \frac{\varphi\left(e^{N\left(r, \frac{1}{f}\right)}\right)}{-\log(1-r)}, \qquad \lambda_{\varphi}^{1}(f) = \limsup_{r \to 1^-} \frac{\varphi\left(N\left(r, \frac{1}{f}\right)\right)}{-\log(1-r)}.$$

where $N(r, f)$ is the integrated Counting function (see [4]). If we replace the lim sup by the lim inf, we obtain the lower φ-convergence exponents of f, $\underline{\lambda}_{\varphi}^{0}(f)$ and $\underline{\lambda}_{\varphi}^{1}(f)$.

Theorem 4.
Let $A_0, \ldots, A_{k-1}$ be analytic functions in Δ and let $\varphi \in \Phi$. Assume that

$$\max\left\{\sigma_{\varphi}^{0}(A_j) : j = 1, \ldots, k-1\right\} \leq \mu_{\varphi}^{0}(A_0) = \mu_0, \ (0 < \mu_0 < +\infty)$$

and

$$\max\left\{\tau_{\varphi}^{0}(A_j) : \sigma_{\varphi}^{0}(A_j) = \mu_{\varphi}^{0}(A_0); j \neq 0\right\} < \underline{\tau}_{\varphi}^{0}(A_0) = \tau_0, \ (0 < \tau_0 < +\infty).$$

Then, every solution $f \not\equiv 0$ of 1 satisfies

$$\underline{\lambda}_{\varphi}^{1}(f - z) = \mu_{\varphi}^{1}(f) = \mu_{\varphi}^{0}(A_0) \leq \sigma_{\varphi}^{1}(f) = \sigma_{\varphi}^{0}(A_0) = \lambda_{\varphi}^{1}(f - z).$$

Open problem
It is interesting to discuss the case when the coefficients of equation 1 are meromorphic functions in the unit disc Δ.

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Growth of solutions of a class of linear differential equations near a singular point

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Abstract: In this paper, we investigate the growth of solutions of the differential equation

$$f'' + A(z)\exp\left\{\frac{a}{(z_0 - z)^n}\right\} f' + B(z)\exp\left\{\frac{b}{(z_0 - z)^n}\right\} f = 0,$$

where $A(z), B(z)$ are analytic functions in the closed complex plane except at z_0 and a, b are complex constants such that $ab \neq 0$ and $a = cb$ $(c > 1)$. Another case has been studied for higher order linear differential equations with analytic coefficients having the same order near a finite singular point.

Keywords: Linear differential equations; local growth of solutions; analytic function; finite singular point.

The linear differential equation

$$f'' + A(z)e^{az}f' + B(z)e^{bz}f = 0, \tag{1}$$

where $A(z)$ and $B(z)$ are entire functions, is investigated by many authors; see for example [8, 9]. In [13], Kwon proved that if $ab \neq 0$ and $\arg a \neq \arg b$ or $a = cb$ with $0 < c < 1$, then every solution $f(z) \not\equiv 0$ of (1) is of infinite order; after, Chen completed the case $c > 1$ in [9]. In 2012, Hamouda proved results similar to (1) in the unit disc concerning the differential equation

$$f'' + A(z)e^{\frac{a}{(z_0 - z)^\mu}} f' + B(z)e^{\frac{b}{(z_0 - z)^\mu}} f = 0, \tag{2}$$

where $\mu > 0$ and $\arg a \neq \arg b$ or $a = cb$ $(0 < c < 1)$, see [6]. After that, Fettouch and Hamouda proved the following two results.

Theorem A. [2] Let z_0, a, b be complex constants such that $\arg a \neq \arg b$ or $a = cb$ $(0 < c < 1)$ and n be a positive integer. Let $A(z), B(z) \not\equiv 0$ be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$ with $\max\{\sigma(A, z_0), \sigma(B, z_0)\} < n$. Then every solution $f(z) \not\equiv 0$ of the differential equation

$$f'' + A(z)\exp\left\{\frac{a}{(z_0 - z)^n}\right\} f' + B(z)\exp\left\{\frac{b}{(z_0 - z)^n}\right\} f = 0,$$

satisfies $\sigma(f, z_0) = \infty$, with $\sigma_2(f, z_0) = n$.

Theorem B. [2] Let $A_0(z) \not\equiv 0, A_1(z), ..., A_{k-1}(z)$ be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfying $\max\{\sigma(A_j, z_0) : j \neq 0\} < \sigma(A_0, z_0)$. Then, every solution $f(z) \not\equiv 0$ of the differential equation

$$f^{(k)} + A_{k-1}(z) f^{(k-1)} + ... + A_1(z) f' + A_0(z) f = 0. \tag{3}$$

satisfies $\sigma(f, z_0) = \infty$, with $\sigma_2(f, z_0) = \sigma(A_0, z_0)$.

In this paper, we will investigate the case $c > 1$ to complete the remaining case in Theorem A, in the following two results.

Theorem C. [1] Let $n \in \mathbb{N} \setminus \{0\}$, $A(z) \not\equiv 0$, $B(z) \not\equiv 0$ be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$ such that $\max\{\sigma(A, z_0), \sigma(B, z_0)\} < n$. Let a, b be complex constants such that $ab \neq 0$ and $a = cb$, $c > 1$. Then every solution $f(z) \not\equiv 0$ of the differential equation

$$f'' + A(z)\exp\left\{\frac{a}{(z_0 - z)^n}\right\} f' + B(z)\exp\left\{\frac{b}{(z_0 - z)^n}\right\} f = 0, \tag{4}$$

that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfies $\sigma(f, z_0) = \infty$.

Theorem D. [1] Let $n \in \mathbb{N} \setminus \{0\}$, $A(z) \not\equiv 0$, $B(z) \not\equiv 0$ be polynomials. Let a, b be complex constants such that $a = cb$, $c > 1$. Then every solution $f(z) \not\equiv 0$ of the differential equation

$$f'' + A\left(\frac{1}{z_0 - z}\right)\exp\left\{\frac{a}{(z_0 - z)^n}\right\} f' + B\left(\frac{1}{z_0 - z}\right)\exp\left\{\frac{b}{(z_0 - z)^n}\right\} f = 0 \tag{5}$$

that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfies $\sigma(f, z_0) = \infty$, with $\sigma_2(f, z_0) = n$.

In the following result, we will improve Theorem B by studying the case when
$\max\{\sigma(A_j, z_0) : j \neq 0\} \leq \sigma(A_0, z_0)$.

Theorem E. [1] Let $A_0(z) \not\equiv 0$, $A_1(z), ..., A_{k-1}(z)$ be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfying the following conditions
i) $0 < \sigma(A_j, z_0) \leq \sigma(A_0, z_0) < \infty, j = 1, ..., k-1$;
ii) $\max\{\tau_M(A_j, z_0) : \sigma(A_j, z_0) = \sigma(A_0, z_0)\} < \tau_M(A_0, z_0)$.
Then, every solution $f(z) \not\equiv 0$ of (3) that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$, satisfies $\sigma(f, z_0) = \infty$, with $\sigma_2(f, z_0) = \sigma(A_0, z_0)$.

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On some stability conditions for fractional-order dynamical systems of order $\alpha \in [0, 2)$ and their applications to some population dynamic models

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Abstract: In this paper some stability criterion have been extended to fractional systems of order $\alpha \in [0, 2)$. Stability diagram and phase portraits classification in the (τ, Δ)-plane for planer fractional-order system are reported. Finally some numerical examples from population dynamics are employed to illustrate the obtained theoretical results.

Keywords: Fractional system, Routh-Hurwitz criterion,p Stability, Population dynamics.

In the past few decades, fractional calculus theory has been improved significantly and has been successfully applied to various research fields. (see for example [1, 2]).

In this paper, we consider the standard fractional differential equation:

$$D^{\alpha}x(t) = f(x(t)), \qquad \alpha \in [0, 2), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and D^{α} is the Caputo derivative operator defined as follows:

$$D^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau. \tag{2}$$

Where, m is the first integer greater than α.

Stability analysis of fractional differential equations was investigated by Matignon who introduced the following theorem when the order of derivative is between 0 and 1.

Theorem. [3] The autonomous system:

$$D^{\alpha}x(t) = Ax(t) \text{ with } x(t_0) = x_0, \tag{3}$$

is asymptotically stable if and only if

$$|\arg(spec(A))| > \frac{\alpha\pi}{2}, \tag{4}$$

where $\alpha \in [0, 1)$, arg(.) is the principal argument of a given complex number and $spec(A)$ is the spectrum (set of all eigenvalues) of A.

In a recent paper [4], the authors derived some *Routh-Hurwitz conditions* of the dynamical systems involving the Caputo fractional derivative which guarantee that all roots of the characteristic polynomial obtained from the linearization process are located inside the Matignon stability sector when the order of derivative is between 0 and 1.
For $0 < \alpha < 2$, an extension of Matignon's theorem is reported in [5]. In this paper we extend the *Routh-Hurwitz conditions* to fractional order systems of order $\alpha \in [0, 2)$, and we report the stability diagram and phase portraits classification in the $(\tau, \Delta)-$plane for planer fractional-order systems. We use these results to investigate the stability properties of some population models.

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STUDY THE SEIQRDP MODEL OF COVID-19 IN ALGERIA

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Abstract: In this talk, we discuss the local existence and uniqueness of the solution for SEIQRDP model of Covid-19 by using fixed point theory in the setting of partial metric spaces. At the end, the model is demonstrated with appropriate numerical and graphical description with case of Algeria.

Keywords: COVID-19, fixed point, partial metric space.

Introduction

All of the studies in modeling the spread of COVID-19 have considered as ordinary differential equations, and then is very important to study the mathematical models of infectious diseases for a better understanding of their evaluation, existence, stability, and control. The aim of the paper is to provide the local existence of solutions for the following SEIQRDP model of corona-virus [3]

$$\begin{cases} S'(t) = -\beta \frac{S(t)I(t)}{N_{pop}} - \alpha S(t) \\ E'(t) = \beta \frac{S(t)I(t)}{N_{pop}} - \gamma E(t) \\ I'(t) = \gamma E(t) - \delta I(t) \\ Q'(t) = \delta I(t) - \lambda(t)Q(t) - \kappa(t)Q(t) \\ R'(t) = \lambda(t)Q(t) \\ D'(t) = \kappa(t)Q(t) \\ P'(t) = \alpha S(t) \end{cases} \tag{1}$$

subject to initial conditions

$$K = \begin{pmatrix} S(0) \\ E(0) \\ I(0) \\ Q(0) \\ R(0) \\ D(0) \\ P(0) \end{pmatrix} = \begin{pmatrix} S_0 \\ E_0 \\ I_0 \\ Q_0 \\ R_0 \\ D_0 \\ P_0 \end{pmatrix} \geq 0_{\mathbb{R}^7}.$$

Where N_{pop} is the total population defined by $N_{pop} = S + E + I + Q + R + D + P$ and

$$\lambda(t) = \frac{\lambda_0}{1 + \exp(-\lambda_1(t - \lambda_2))}, \quad \kappa(t) = \frac{\kappa_0}{\exp(-\kappa_1(t - \kappa_2)) + \exp(\kappa_1(t - \kappa_2))}. \tag{2}$$

The population is assumed constant. The needed fixe point theorem reads as follow:

Parameters	The physical interpretation
$\mathbf{S}$(t)	Susceptible cases
$\mathbf{E}$(t)	Exposed cases
$\mathbf{I}$(t)	Infected cases
$\mathbf{Q}$(t)	Quarantined cases
$\mathbf{R}$(t)	Recovered cases
$\mathbf{D}$(t)	Dead cases
$\mathbf{P}$(t)	Insusceptible or protected cases
α	the protection rate
β	the infection rate
γ^{-1}	the average latent time
δ	the rate at which infectious people enter in quarantine
$\lambda(t)$	time-dependant recovery rate
$\kappa(t)$	time-dependant mortality rate

Table 1: Description of the parameters used in model (1)

Theorem 1. [2] Let (X, p) be a complete partial metric space. Let $\overline{x} \in X$ and $r > 0$ such that $\phi : \overline{B_p}(\overline{x}, r) \rightarrow C^p(X)$ be a set-valued mapping. Let $\varphi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a increasing and continuous function such that φ is a Bianchini-Grandolfi gauge function on interval J and $\lim_{t \downarrow 0} \varphi(t) = 0$. If there exists $\alpha \in J$ such that the following two conditions hold:

(a) $p(\overline{x}, \phi(\overline{x})) < \alpha$ where $s(\alpha) \leq p(\overline{x}, \overline{x}) + r$

(b) $\delta_p(\phi(x) \cap \overline{B_p}(\overline{x}, r), \phi(y)) \leq \varphi\left(p(x, y)\right) \quad \forall x, y \in \overline{B_p}(\overline{x}, r)$,

then ϕ has a fixed point x^* in $\overline{B_p}(\overline{x}, r)$. If ϕ is a single valued mapping and $p(\overline{x}, \overline{x}) + 2r \in J$, then x^* is the unique fixed point of ϕ in $\overline{B_p}(\overline{x}, r)$.

Main result

Consider a Banach space $\Omega = (C[0, T])^7$, the product space of all continuous real functions defined on $I = [0, T]$, with a norm

$$\|(S, E, I, Q, R, D, P)\|_\Omega = \max_{t \in [0,T]} (|S(t)| + |E(t)| + |I(t)| + |Q(t)| + |R(t)| + |D(t)| + |P(t)|)$$

and the metric associated is $d(u,v) = \|u-v\|_\Omega$ for all $u, v \in \Omega$. Let Ω be endowed with the partial metric [1]

$$p(u,v) = \frac{1}{2}d(u,v) + c = \frac{1}{2}\|u-v\|_\Omega + c \qquad \forall u, v \in \Omega,$$

where $c \geq 0$. We consider the following conditions:

(C1) There exists an increasing and continuous function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ such that φ is a Bianchini-Grandolfi gauge function on interval J and $\lim_{t\downarrow 0} \varphi(t) = 0$

(C2) There exists $\alpha \in J$ such that $\begin{cases} \alpha > c + \frac{1}{2}\|K\|_\Omega, \\ s(\alpha) \leq c + N_{pop}, \end{cases}$

(C3) $|f_i(t,u(t)) - f_i(t,v(t))| \leq \dfrac{2}{7T}(\varphi(\tfrac{1}{2}\|u-v\|_\Omega + c) - c) \quad \forall \begin{cases} t \in I, i \in \{1,\dots,7\} \\ \|u\|_\Omega, \|v\|_\Omega \leq 2N_{pop}. \end{cases}$

Theorem 2. For a fixed $c \geq 0$, suppose that conditions (C1)-(C3) holds. Then (1) has at least one solution $\overline{X} \in \Omega$ such that $\|\overline{X}\|_\Omega \leq 2N_{pop}$. Moreover, if $c + 2N_{pop} \in J$ then the solution is unique.

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